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Solar Power Plant Location Selection Problem Using the ELECTRE-III Method in a Pythagorean Neutrosophic Programming Approach: A Case Study on Green Energy in India

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Abstract

India has set an ambitious target of generating 500 GW of renewable energy capacity from non-fossil fuel sources by 2030. It has also committed, under the United Nations Framework Convention on Climate Change (UNFCCC), to reducing carbon emissions by one billion tonnes by the end of the decade, as announced at the COP26 conference held in Glasgow in November 2022. Researchers are continually searching for inexhaustible and affordable energy sources. Solar energy is among the greenest and cleanest sources of energy. The most important factor in utilizing solar energy is the location of the solar power plant. The main objective of this study is to find the best location for a new solar power plant in a specific region called the Bundelkhand region of Uttar Pradesh in India. Here, we offer an extension of the ELECTRE-III method as a two-phase Pythagorean neutrosophic elimination and choice translating reality (PN-ELECTRE-III) method to adapt with fuzzy, ambiguous, unsure, and indeterminate criteria. The Pythagorean neutrosophic numbers (PNNs) are used by the group decision support system of PN-ELECTRE III to measure the performance of the alternatives. The options are entirely outclassed in the subsequent stage in view of the past stage's evaluations of them. By defining PNN, we describe the technique of indifference threshold, preference threshold, and veto threshold functions, which provide a more stable basis to drop outranking relations. By calculating the concordance credibility, discordance credibility, and net credibility degrees of each alternative, the ranking module of the PN-ELECTRE III approach is made simpler. In order to confirm the applicability of the strategy suggested in this paper, the location selection problem for solar plants is finally solved.

Categories: Expert Systems, Fuzzy Logic, Soft Computing
Keywords: solar power plant, multi-criteria decision-making, pythagorean neutrosophic numbers, pn-electre-iii method, renewable energy

Introduction

With the rise of environmental concerns and the depletion of non-sustainable energy resources, renewable energy sources have become widely recognized. Solar energy is one of the most abundant renewable energy sources. However, the location of the solar plant is critical as it affects the efficiency and profitability of the plant. A suitable site for the solar plant should have high solar irradiance, flat terrain, and adequate space. Consequently, choosing the right location for the solar plant is a crucial task. Decision-making for selecting the suitable location for the solar plant is a multi-criteria decision-making (MCDM) problem [1,2]. The MCDM problem deals with selecting the best alternatives based on multiple criteria. The ELECTRE-III method is a widely used MCDM method that considers multiple criteria and ranks alternatives based on the criteria [3], although the ELECTRE-III method does not consider uncertainty and vagueness in the decision-making process.

Fuzzy set theory is an extension of the traditional crisp set theory by Zadeh [4]. This makes it a useful tool in MCDM problems, where the criteria can frequently be uncertain or imprecise. By giving the logical framework for fuzzy set, whose distinctive feature is to show cryptic data by the dint of membership function, Zadeh [4] made an extraordinary commitment in this area. In order to ranking, Aleskerov et al. [5] presented modified versions of the ELECTRE-III process in a fuzzy environment. Gao et al.'s [6] analysis of the competitiveness of China's Quanzhou port combined the fuzzy-AHP and ELECTRE-III techniques. Kaur and Yadav [7] used spherical fuzzy programming to solve the transportation problem.

In 1986, Atanassov [8] developed the basis for a realistic model with a changed structure, namely, the intuitionistic fuzzy set (IFS), to explain the ambiguous information using satisfaction and dissatisfaction degrees under the given constraints. With the use of the ELECTRE-III approach for intuitionistic fuzzy models, Zhang et al. [9] investigated a case for the selection of the most suitable location for an offshore

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wind power station. IFS theory is a further extension of the traditional fuzzy set theory. It allows elements to have degrees of membership and non-membership that are not necessarily equal to one another, making it a useful tool in dealing with uncertainty in MCDM problems [10]. Further Pythagorean fuzzy subsets, as the extension of IFS, made problems more advanced [11-19].

Neutrosophic set theory (NST) as an extension of the traditional crisp, fuzzy, and IFS theories introduced by Smarandache [20], in 1998. It allows elements to have degrees of membership, degrees of non-membership, and degrees of indeterminacy that are independent of one another, making it a useful tool in dealing with incomplete or inconsistent information in MCDM problems. By introducing a new neutrosophic outranking relation based on the concept of truth-membership, falsity-non-membership, and indeterminacy-membership degrees, the ELECTRE-III method has been further extended to handle neutrosophic sets [21-28].

Recently, Jansi et al. [29] and Rajan and Krishnaswamy [30] introduced the Pythagorean neutrosophic set (PNS), which was a further extension of the NST. It allows elements to have three independent values, such as truth-membership, indeterminacy-membership, and falsity-membership degrees. PNS is a useful and effective tool to handle incomplete or inconsistent information in MCDM problems. The Pythagorean neutrosophic programming (PNP) approach is a generalization of fuzzy set theory and NST that can handle uncertain, incomplete, and inconsistent information in the decision-making process [31]. In the present research problem, we are utilizing the PNP approach with the ELECTRE-III technique, which can improve the decision-making process [32].

In this paper, we propose a two-phase decision-aiding system for the solar plant location problem using the ELECTRE-III method in the PNP approach. The first phase of the proposed system uses the PNP approach to handle uncertain and incomplete information in the decision-making process. In the second phase, the ELECTRE-III method is used to rank the alternatives based on the criteria [3]. The proposed system can provide decision-makers with a set of feasible alternatives and their ranking based on the criteria.

A case study on green energy in the Bundelkhand region of India is conducted to demonstrate the effectiveness of the proposed system. The study considers seven potential sites for the solar plant, and the decision criteria are abundant solar radiation, flat and open land, high land and construction costs, demand for electricity, extreme weather conditions, higher elevation from sea level, proximity to transmission lines, and average dust density to the substation. The results show that the proposed system can provide decision-makers with a set of feasible alternatives and their ranking based on the criteria, which can help decision-makers in selecting the most suitable site for the solar plant.

Motivation behind the current study

The increasing demand for clean and sustainable energy has led to a surge in the installation of solar power plants [2,33,34]. However, identifying the optimal location for a solar plant is a complex problem due to numerous factors such as weather conditions, land availability, infrastructure, and environmental impact. In this context, decision-aiding systems can assist decision-makers in identifying the best location for solar plant installation. This study aims to develop a two-phase decision-aiding system [3] using the ELECTRE-III method in the PNP approach for the solar plant location problem. The proposed method considers the imprecise and uncertain nature of the decision-making process in the solar plant location problem. A contextual analysis on green energy in the Bundelkhand region of India is conducted to demonstrate the applicability and effectiveness of the proposed method.

PNSs are adopted in this study because they provide a higher expressive capability than traditional fuzzy, intuitionistic fuzzy, and Pythagorean fuzzy models. Unlike these earlier frameworks, PNS allows the truth-membership, indeterminacy-membership, and falsity-membership degrees to vary independently, which offers greater flexibility for representing uncertainty, inconsistency, and hesitation commonly observed in real-world solar site assessment. This makes PNS particularly suitable for complex multi-criteria group decision-making (MCGDM) where expert judgments are imprecise or conflicting.

The consequences of this study can help policymakers and partners in pursuing informed choices in regard to the optimal location for solar plant establishment. This research also contributes to the literature on decision-making under uncertainty using PN-ELECTRE-III method.

Objectives of the current investigation

The objective of this investigation is to propose a two-phase decision-aiding system for selecting suitable locations for solar plants in the Bundelkhand region of India, using the PN-ELECTRE III method. The proposed system aims to consider various factors such as economic, social, environmental, and technical aspects to select the optimal location for solar plants in Bundelkhand. The first phase of the proposed system involves the development of a PN-ELECTRE III method to evaluate the suitability of potential locations for solar plants. The second phase involves the development of a decision-aiding system to

prioritize the locations based on the evaluation results of the first phase. The proposed system will take into account the uncertainties and imprecision in decision-making using PNP. The study will contribute to the field of decision-making in renewable energy by developing a comprehensive decision-aiding system that considers multiple criteria and uncertain information [10]. The proposed system will also provide valuable insights into the selection of optimal locations for solar plants in seven districts, Banda, Chitrakoot, Hamirpur, Jalaun, Jhansi, Lalitpur, and Mahoba, of the Bundelkhand region and help policymakers and investors to make informed decisions.

Related research work

The proposed approach builds on the existing literature on decision-making methods and multicriteria decision-making in particular. The ELECTRE III method has been widely used in previous research for decision-making in different contexts, such as supply chain management, transportation, and environmental management. Moreover, the PNP approach has been used in previous research to address decision-making problems under uncertainty. Some studies [1,9,35] applied the PNP approach to rank different photovoltaic power plant locations in China.

Materials And Methods

Preliminaries

In this segment, some fundamental preliminary concepts of neutrosophic sets, single-valued neutrosophic sets, PNS, and PNN are briefly presented, which will enable the conversation in the following sections.

Definition 1. [1,20]. Let U be a non-empty set. An neutrosophic set P on U, containing $\phi_P(u)$ as the degree of membership, $\varphi_P(u)$ as the degree of indeterminacy, and $\gamma_P(u)$ as the degree of non-membership, is defined as:

$$P = \{\langle u, \phi_P(u), \varphi_P(u), \gamma_P(u)\rangle : u \in U\}$$

where $\phi_P(u), \varphi_P(u), \gamma_P(u) \in \left]^{-}0, 1^{+}\right[$, such that

$^{-}0 \leq \phi_P(u) + \varphi_P(u) + \gamma_P(u) \leq 3^{+}$, for all $u \in U$

Definition 2. [36,37]. A single-valued neutrosophic set P on a non-empty universal set U is defined as:

$$P = \{\langle u, \phi_P(u), \varphi_P(u), \gamma_P(u)\rangle : u \in U\}$$

where $\phi_P(u), \varphi_P(u), \gamma_P(u) \in [0, 1]$, such that $0 \leq \phi_P(u) + \varphi_P(u) + \gamma_P(u) \leq 3$ for all $u \in U$; $\phi_P(u)$ is the membership degree function, $\varphi_P(u)$ is the indeterminacy degree function, and $\gamma_P(u)$ is the non-membership degree function.

Definition 3. [29,30,38]. Let U be a non-empty universal set of discourse. A PNS P on U is defined as:

$$P = \{\langle u, \phi_P(u), \varphi_P(u), \gamma_P(u)\rangle : u \in U\}$$

where $\phi_P(u), \varphi_P(u), \gamma_P(u) \in [0, 1]$, such that

$0 \leq (\phi_P(u))^2 + (\varphi_P(u))^2 + (\gamma_P(u))^2 \leq 2$ for all $u \in U$, and $\phi_P(u)$ is the membership degree function, $\varphi_P(u)$ is the indeterminacy degree function, and $\gamma_P(u)$ is the non-membership degree function. Here, truth $\phi_P(u)$ and falsity $\gamma_P(u)$ are dependent components, and indeterminacy $\varphi_P(u)$ is an independent component. The triplet $P = (\phi_P(u), \varphi_P(u), \gamma_P(u))$ is called a Pythagorean neutrosophic number (PNN). For convenience, we represent a PNN $P = (\phi_P(u), \varphi_P(u), \gamma_P(u)) = (\phi_P, \varphi_P, \gamma_P)$ throughout in this article.

Definition 4. [29]. (Operations) Let three PNNs be given as $P = (\phi_P, \varphi_P, \gamma_P)$, $P_1 = (\phi_{P_1}, \varphi_{P_1}, \gamma_{P_1})$, and $P_2 = (\phi_{P_2}, \varphi_{P_2}, \gamma_{P_2})$. Then, the elementary operations on these PNNs are defined as follows:

Complement: $P^C = (\gamma_P, \varphi_P, \phi_P)$

Union: $P_1 \cup P_2 = (\max\{\phi_{P_1}, \phi_{P_2}\}, \min\{\varphi_{P_1}, \varphi_{P_2}\}, \min\{\gamma_{P_1}, \gamma_{P_2}\})$

Intersection: $P_1 \cap P_2 = (\min\{\phi_{P_1}, \phi_{P_2}\}, \max\{\varphi_{P_1}, \varphi_{P_2}\}, \max\{\gamma_{P_1}, \gamma_{P_2}\})$

Addition: $P_1 \oplus P_2 = \left(\sqrt{\phi_{P_1}^2 + \phi_{P_2}^2 - \phi_{P_1}^2 \cdot \phi_{P_2}^2}, \varphi_{P_1} \cdot \varphi_{P_2}, \gamma_{P_1} \cdot \gamma_{P_2}\right)$

Multiplication:

$$P_1 \otimes P_2 = \left(\phi_{P_1} \cdot \phi_{P_2}, \sqrt{\varphi_{P_1}^2 + \varphi_{P_2}^2 - \varphi_{P_1}^2 \cdot \varphi_{P_2}^2}, \sqrt{\gamma_{P_1}^2 + \gamma_{P_2}^2 - \gamma_{P_1}^2 \cdot \gamma_{P_2}^2}\right)$$

Scalar multiplication: $r \cdot P = \left(\sqrt{1-(1-\phi_P^2)^r}, \varphi_P^r, \gamma_P^r\right)$, where $r > 0$

Exponentiation: $P^r = \left(\phi_P^r, \sqrt{1-(1-\varphi_P^2)^r}, \sqrt{1-(1-\gamma_P^2)^r}\right)$, where $r > 0$

Definition 5. [39,40]. (De-Neutrosophication)

Score function: $s(P) = \phi_P^2 - \varphi_P^2 - \gamma_P^2$

Accuracy function: $a(P) = \phi_P^2 + \varphi_P^2 + \gamma_P^2$

Normalized Euclidean distance:
$d(P_1, P_2) = \sqrt{(\phi_{P_1}^2 - \phi_{P_2}^2)^2 + (\varphi_{P_1}^2 - \varphi_{P_2}^2)^2 + (\gamma_{P_1}^2 - \gamma_{P_2}^2)^2}$

Definition 6. [3,40]. (Comparison)

If $s(P_1) > s(P_2)$, then $P_1 \succ P_2$ (i.e., P_1 is superior to P_2)

If $s(P_1) = s(P_2)$, then:

- If $a(P_1) > a(P_2)$, then $P_1 \succ P_2$

- If $a(P_1) = a(P_2)$, then $P_1 \sim P_2$

Definition 7. [3]. (Aggregation): Let $P_i = (\phi_{P_i}, \varphi_{P_i}, \gamma_{P_i})$, $w = (w_1, w_2, \ldots, w_n)$ such that $\sum_{i=1}^{n} w_i = 1$, then

Pythagorean neutrosophic weighted average (PNWA) operator:

$$PNWA_w(P_1, P_2, \ldots, P_n) = w_1 P_1 \oplus w_2 P_2 \oplus \cdots \oplus w_n P_n$$

$$= \left(\sqrt{1 - \prod_{i=1}^{n}(1-\phi_{P_i}^2)^{w_i}}, \prod_{i=1}^{n}(\varphi_{P_i})^{w_i}, \prod_{i=1}^{n}(\gamma_{P_i})^{w_i}\right) \quad (1)$$

Pythagorean neutrosophic weighted geometric (PNWG) operator:

$$PNWG_w(P_1, P_2, \ldots, P_n) = w_1 P_1 \otimes w_2 P_2 \otimes \cdots \otimes w_n P_n$$

$$= \left(\prod_{i=1}^{n}(\phi_{P_i})^{w_i}, \sqrt{1 - \prod_{i=1}^{n}(1-\varphi_{P_i}^2)^{w_i}}, \sqrt{1 - \prod_{i=1}^{n}(1-\gamma_{P_i}^2)^{w_i}}\right) \quad (2)$$

ELECTRE-III method

Let $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_a, \ldots, \lambda_f\}$ be a set of available *f* alternatives, and $X = \{\chi_1, \chi_2, \chi_3, \ldots, \chi_b, \ldots, \chi_g\}$ be a set of *g* criteria corresponding to each alternative for an MCGDM problem. A group $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_t, \ldots, \varepsilon_h\}$ of *h* experts or decision-makers allocates the feasibility, performance, or evaluation information of alternative $\lambda_a \in \Lambda$ with respect to criterion $\chi_b \in X$ as $\chi_b(\lambda_a)$. The more the alternative satisfies the criterion, the lower or higher the value of $\chi_b(\lambda_a)$, which depends on whether the objective is to minimize or to maximize the criterion χ_b. Subsequently, the performance or feasibility information of an alternative λ_a, on the basis of multiple criteria, will be represented by the vector $\chi(\lambda_a) = \{\chi_1(\lambda_a), \chi_2(\lambda_a), \ldots, \chi_g(\lambda_a)\}$. As all the criteria may have different importance depending on the objective of the MCGDM problem, the criteria weight vector will be denoted by $w = \{w_1, w_2, w_3, \ldots, w_b, \ldots, w_g\}$ such that $\sum_{b=1}^{g} w_b = 1$. Similarly, the expert importance weight vector will be

$$\ell = \{\ell_1, \ell_2, \ell_3, \ldots, \ell_t, \ldots, \ell_h\} \text{ such that } \sum_{t=1}^{h} \ell_t = 1.$$

The ranking process of the ELECTRE III method consists of two modules [3,6,35,41,42]. After the performance evaluation of alternatives by evaluators or DMs over multiple criteria, the establishment of preference and indifference threshold functions, the determination of concordance and discordance indices, and ultimately the revelation of the credibility index come under the first module in the formation of an outranking connection. Using outranking relations to deduce a comprehensive feasibility ranking of alternatives constitutes the second module.

Module I: Developing Outranking Relations

In the MCGDM process, an alternative λ_1 outranks another alternative λ_2, represented by $\lambda_1 S \lambda_2$, if there is sufficient evidence to believe that λ_1 is at least as good as λ_2 and there are no compelling counterarguments. The ELECTRE method's fundamental tenet is to establish a preference relation - often referred to as an outranking relation - among the acts assessed across a number of criteria. The basis for establishing the outranking relation $\lambda_1 S \lambda_2$ is provided by the credibility index, which is the degree of outranking. The concordance index and discordance index, computed for each criterion $\chi_b \in X$, are used to calculate the degree of credibility.

Erection of threshold functions: The ELECTRE III model's assessment technique includes establishing indifference threshold function, preference threshold function, and veto threshold function for disclosing concordance and discordance indices, determining the degree of credibility, and ranking the alternatives. Let $q(\chi_b)$ be the indifference threshold function and $p(\chi_b)$ be the preference threshold function corresponding to criterion χ_b.

Let for any two given alternatives $\lambda_1, \lambda_2 \in \Lambda$, if $\chi(\lambda_1) \geq \chi(\lambda_2)$, then:

$$\chi(\lambda_1) \succ \chi(\lambda_2) + p(\chi(\lambda_2)) \Leftrightarrow \lambda_1 P \lambda_2 \tag{3}$$

$$\chi(\lambda_2) + q(\chi(\lambda_2)) \prec \chi(\lambda_1) \prec \chi(\lambda_2) + p(\chi(\lambda_2)) \Leftrightarrow \lambda_1 Q \lambda_2 \tag{4}$$

$$\chi(\lambda_2) \prec \chi(\lambda_1) \prec \chi(\lambda_2) + q(\chi(\lambda_2)) \Leftrightarrow \lambda_1 I \lambda_2 \tag{5}$$

where $\chi(\lambda)$ is the criterion score value of the alternative λ, and *P* signifies strong preference, *Q* weak preference, and *I* indifference.

Calculating the concordance index of the assertion $\lambda_1 S \lambda_2$: For each pair of alternatives, the comprehensive concordance index is defined as:

$$\Delta(\lambda_1, \lambda_2) = \sum_{b=1}^{g} w_b \Delta_b(\lambda_1, \lambda_2) \tag{6}$$

where w_b represents the weight of the b^{th} criterion and $\Delta_b(\lambda_1, \lambda_2)$ represents the partial concordance index over criterion χ_b, calculated as:

$$\Delta_b(\lambda_1, \lambda_2) = \begin{cases} 0, & \text{if } \chi_b(\lambda_2) - \chi_b(\lambda_1) > p(\chi_b) \\ 1, & \text{if } \chi_b(\lambda_2) - \chi_b(\lambda_1) \leq q(\chi_b) \\ \dfrac{p(\chi_b) - [\chi_b(\lambda_2) - \chi_b(\lambda_1)]}{p(\chi_b) - q(\chi_b)}, & \text{otherwise} \end{cases} \tag{7}$$

Thus, $0 \leq \Delta_b(\lambda_1, \lambda_2) \leq 1$.

Calculating the discordance index of the assertion $\lambda_1 S \lambda_2$: For each criterion, the discordance index $\nabla_b(\lambda_1, \lambda_2)$ is calculated as:

$$\nabla_b(\lambda_1, \lambda_2) = \begin{cases} 0, & \text{if } \chi_b(\lambda_2) - \chi_b(\lambda_1) \leq p(\chi_b) \\ 1, & \text{if } \chi_b(\lambda_2) - \chi_b(\lambda_1) > v(\chi_b) \\ \dfrac{[\chi_b(\lambda_2) - \chi_b(\lambda_1)] - p(\chi_b)}{v(\chi_b) - p(\chi_b)}, & \text{otherwise} \end{cases} \tag{8}$$

Thus, $0 \leq \nabla_b(\lambda_1, \lambda_2) \leq 1$.

Disclosure of credibility index: The credibility index, denoted by $\pi(\lambda_1, \lambda_2)$, is used to determine the degree of the outranking relation $\lambda_1 S \lambda_2$ and is defined as:

$$\pi(\lambda_1, \lambda_2) = \begin{cases} \Delta(\lambda_1, \lambda_2), & \text{if } \nabla_b(\lambda_1, \lambda_2) \leq \Delta(\lambda_1, \lambda_2),\ \forall b \in \tau \\ \Delta(\lambda_1, \lambda_2) \times \prod\limits_{b \in \tau'} \frac{1 - \nabla_b(\lambda_1, \lambda_2)}{1 - \Delta(\lambda_1, \lambda_2)}, & \text{otherwise} \end{cases} \tag{9}$$

where $\tau' = \{b \in \tau : \nabla_b(\lambda_1, \lambda_2) > \Delta(\lambda_1, \lambda_2)\}$.

Module II: The Exploitation of Outranking Relations

The standard ranking approach of ELECTRE III employs a structured algorithm via two intermediate ranking procedures: descending distillation (ranking from best to worst) and ascending distillation (ranking from worst to best). Alternatively, a new ranking method based on the concepts of concordance credibility degree, discordance credibility degree, and net credibility degree is used, as proposed by Li and Wang [41].

For each alternative, the concordance credibility degree is defined as:

$$\eta^{+}(\lambda_a) = \sum_{\lambda_b \in \Lambda} \pi(\lambda_a, \lambda_b), \quad \forall \lambda_a \in \Lambda \tag{10}$$

For each alternative, the discordance credibility degree is defined as:

$$\eta^{-}(\lambda_a) = \sum_{\lambda_b \in \Lambda} \pi(\lambda_b, \lambda_a), \quad \forall \lambda_a \in \Lambda \tag{11}$$

For each alternative, the net credibility degree is defined as:

$$\eta(\lambda_a) = \eta^{+}(\lambda_a) - \eta^{-}(\lambda_a), \quad \forall \lambda_a \in \Lambda \tag{12}$$

The attractiveness of an alternative λ_a increases with the value of the net credibility degree $\eta(\lambda_a)$. Therefore, alternatives can be ranked in decreasing order of $\eta(\lambda_a)$.

Two-phase Pythagorean neutrosophic ELECTRE-III method: Algorithm

In this part, the two-phase PN-ELECTRE III group decision support system is created by combining the PNSs and the traditional ELECTRE-III approach. In the Pythagorean neutrosophic environment, consider an MCGDM problem,where $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, ..., \lambda_a, ..., \lambda_f\}$ is a set of *f* alternatives, and $X = \{\chi_1, \chi_2, \chi_3, ..., \chi_b, ..., \chi_g\}$ is a set of *g* criteria assigned to each alternative. A group $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, ..., \varepsilon_t, ..., \varepsilon_h\}$ of *h* experts or decision makers (DMs) provides feasibility information for each alternative $\lambda_a \in \Lambda$ with respect to criterion $\chi_b \in X$, denoted as $\chi_b(\lambda_a)$.

Each criterion can have a different level of importance. Let the criteria weight vector be $w = \{w_1, w_2, w_3, ..., w_b, ..., w_g\}$ such that $\sum_{b=1}^{g} w_b = 1$. Similarly, let the expert weight vector be $\ell = \{\ell_1, \ell_2, \ell_3, ..., \ell_t, ..., \ell_h\}$, such that $\sum_{t=1}^{h} \ell_t = 1$. Let the subscript set of criteria be $\tau = \{1, 2, 3, ..., g\}$.

Phase I: Pythagorean Neutrosophic Evaluation Phase

Step 1. First, establish linguistic variables corresponding to PNNs for the evaluation of feasibility ratings of alternatives and threshold functions, importance weights of criteria, and importance weights of experts.

Step 2. Weight of the t^{th} expert can be determined by the following:

$$\ell_t = \frac{\phi_t + \gamma_t\left(\frac{\phi_t}{\phi_t + \varphi_t}\right)}{\sum_{t=1}^{h}\left(\phi_t + \gamma_t\left(\frac{\phi_t}{\phi_t + \varphi_t}\right)\right)} \tag{13}$$

where ℓ_t satisfies the normalization condition $\sum_{t=1}^{h} \ell_t = 1$.

Step 3. Systematically assess each alternative $\lambda_a \ (a = 1, 2, 3, ..., f)$ with respect to each criterion $\chi_b \ (b = 1, 2, 3, ..., g)$. Each expert $\varepsilon_t \ (t = 1, 2, 3, ..., h)$ provides evaluation information in the form of a Pythagorean neutrosophic decision matrix (PNDM) $M^{(t)} = [M_{ab}^{(t)}]_{f \times g}$, as shown in Matrix 1. Here, $M_{ab}^{(t)} = \left(\phi_{M_a}^{(t)}(\chi_b), \varphi_{M_a}^{(t)}(\chi_b), \gamma_{M_a}^{(t)}(\chi_b)\right)$ is the PNN allocated by expert ε_t, where, $\phi_{M_a}^{(t)}(\chi_b)$ is the membership degree, $\varphi_{M_a}^{(t)}(\chi_b)$ is the indeterminacy degree, and $\gamma_{M_a}^{(t)}(\chi_b)$ is the non-membership degree. Here, the membership $\phi_{M_a}^{(t)}(\chi_b)$ and non-membership $\gamma_{M_a}^{(t)}(\chi_b)$ are dependent components, while the indeterminacy $\varphi_{M_a}^{(t)}(\chi_b)$ is considered independent.

Matrix 1: PNDM by Experts

$M^{(t)}$	χ_1	χ_2	$\cdots$	χ_g
λ_1	$\left(\phi^{(t)}_{M_1}(\chi_1), \varphi^{(t)}_{M_1}(\chi_1), \gamma^{(t)}_{M_1}(\chi_1)\right)$	$\left(\phi^{(t)}_{M_1}(\chi_2), \varphi^{(t)}_{M_1}(\chi_2), \gamma^{(t)}_{M_1}(\chi_2)\right)$	$\cdots$	$\left(\phi^{(t)}_{M_1}(\chi_g), \varphi^{(t)}_{M_1}(\chi_g), \gamma^{(t)}_{M_1}(\chi_g)\right)$
λ_2	$\left(\phi^{(t)}_{M_2}(\chi_1), \varphi^{(t)}_{M_2}(\chi_1), \gamma^{(t)}_{M_2}(\chi_1)\right)$	$\left(\phi^{(t)}_{M_2}(\chi_2), \varphi^{(t)}_{M_2}(\chi_2), \gamma^{(t)}_{M_2}(\chi_2)\right)$	$\cdots$	$\left(\phi^{(t)}_{M_2}(\chi_g), \varphi^{(t)}_{M_2}(\chi_g), \gamma^{(t)}_{M_2}(\chi_g)\right)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
λ_f	$\left(\phi^{(t)}_{M_f}(\chi_1), \varphi^{(t)}_{M_f}(\chi_1), \gamma^{(t)}_{M_f}(\chi_1)\right)$	$\left(\phi^{(t)}_{M_f}(\chi_2), \varphi^{(t)}_{M_f}(\chi_2), \gamma^{(t)}_{M_f}(\chi_2)\right)$	$\cdots$	$\left(\phi^{(t)}_{M_f}(\chi_g), \varphi^{(t)}_{M_f}(\chi_g), \gamma^{(t)}_{M_f}(\chi_g)\right)$

Step 4. The distinct opinions of DMs or experts are combined into a collective opinion to construct the aggregated PNDM matrix $\ddot{M} = \left(\ddot{M}_{ab}\right)_{f\times g}$ using the PNWA operator [39,41]. Let $M^{(t)} = \left(M^{(t)}_{ab}\right)_{f\times g}$ be the PNDM provided by expert ε_t. Then,

$$\begin{aligned}\ddot{M}_{ab} &= PNWA_\ell\left(M^{(1)}_{ab}, M^{(2)}_{ab}, ..., M^{(h)}_{ab}\right)\\ &= \ell_1 M^{(1)}_{ab} \oplus \ell_2 M^{(2)}_{ab} \oplus \ell_3 M^{(3)}_{ab} \oplus \cdots \oplus \ell_h M^{(h)}_{ab}\\ &= \left(\sqrt{1-\prod_{t=1}^{h}\left(1-\left(\phi^{(t)}_{ab}\right)^2\right)^{\ell_t}}, \prod_{t=1}^{h}\left(\varphi^{(t)}_{ab}\right)^{\ell_t}, \prod_{t=1}^{h}\left(\gamma^{(t)}_{ab}\right)^{\ell_t}\right)\end{aligned} \tag{14}$$

where $\ddot{M}_{ab} = \left(\phi_{\ddot{M}_a}(\chi_b), \varphi_{\ddot{M}_a}(\chi_b), \gamma_{\ddot{M}_a}(\chi_b)\right)$ for $a = 1, 2, ..., f$ and $b = 1, 2, ..., g$. Consequently, the aggregated matrix $\ddot{M}$ is shown in Matrix 2.

Matrix 2: Aggregated PNDM

$\ddot{M}$	χ_1	χ_2	$\cdots$	χ_g
λ_1	$\left(\phi_{\ddot{M}_1}(\chi_1), \varphi_{\ddot{M}_1}(\chi_1), \gamma_{\ddot{M}_1}(\chi_1)\right)$	$\left(\phi_{\ddot{M}_1}(\chi_2), \varphi_{\ddot{M}_1}(\chi_2), \gamma_{\ddot{M}_1}(\chi_2)\right)$	$\cdots$	$\left(\phi_{\ddot{M}_1}(\chi_g), \varphi_{\ddot{M}_1}(\chi_g), \gamma_{\ddot{M}_1}(\chi_g)\right)$
λ_2	$\left(\phi_{\ddot{M}_2}(\chi_1), \varphi_{\ddot{M}_2}(\chi_1), \gamma_{\ddot{M}_2}(\chi_1)\right)$	$\left(\phi_{\ddot{M}_2}(\chi_2), \varphi_{\ddot{M}_2}(\chi_2), \gamma_{\ddot{M}_2}(\chi_2)\right)$	$\cdots$	$\left(\phi_{\ddot{M}_2}(\chi_g), \varphi_{\ddot{M}_2}(\chi_g), \gamma_{\ddot{M}_2}(\chi_g)\right)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
λ_f	$\left(\phi_{\ddot{M}_f}(\chi_1), \varphi_{\ddot{M}_f}(\chi_1), \gamma_{\ddot{M}_f}(\chi_1)\right)$	$\left(\phi_{\ddot{M}_f}(\chi_2), \varphi_{\ddot{M}_f}(\chi_2), \gamma_{\ddot{M}_f}(\chi_2)\right)$	$\cdots$	$\left(\phi_{\ddot{M}_f}(\chi_g), \varphi_{\ddot{M}_f}(\chi_g), \gamma_{\ddot{M}_f}(\chi_g)\right)$

Step 5. Let X_B and X_C represent the corresponding collections of criteria that are of the benefit-type and cost-type. The aggregated PNDM, $\ddot{M} = \left(\ddot{M}_{ab}\right)_{f\times g}$, can be converted into the normalized aggregated PNDM, $M = (M_{ab})_{f\times g}$, which displays the evaluation information of each alternative with respect to each benefit or cost criterion in standard form for further calculations. The PNN for M_{ab} can be described as follows:

$$M_{ab} = (\phi_{M_a}(\chi_b), \varphi_{M_a}(\chi_b), \gamma_{M_a}(\chi_b)) = \begin{cases} \ddot{M}_{ab} = \left(\phi_{\ddot{M}_a}(\chi_b), \varphi_{\ddot{M}_a}(\chi_b), \gamma_{\ddot{M}_a}(\chi_b)\right), & \text{if } \chi_b \in \mathrm{X}_B \\ \left(\ddot{M}_{ab}\right)^C = \left(\gamma_{\ddot{M}_a}(\chi_b), \varphi_{\ddot{M}_a}(\chi_b), \phi_{\ddot{M}_a}(\chi_b)\right), & \text{if } \chi_b \in \mathrm{X}_C \end{cases} \tag{15}$$

The following Matrix 3 demonstrates how the matrix *M* is constructed.

Matrix 3: Normalized Aggregated PNDM

M	χ_1	χ_2	$\cdots$	χ_g
λ_1	$(\phi_{M_1}(\chi_1), \varphi_{M_1}(\chi_1), \gamma_{M_1}(\chi_1))$	$(\phi_{M_1}(\chi_2), \varphi_{M_1}(\chi_2), \gamma_{M_1}(\chi_2))$	$\cdots$	$(\phi_{M_1}(\chi_g), \varphi_{M_1}(\chi_g), \gamma_{M_1}(\chi_g))$
λ_2	$(\phi_{M_2}(\chi_1), \varphi_{M_2}(\chi_1), \gamma_{M_2}(\chi_1))$	$(\phi_{M_2}(\chi_2), \varphi_{M_2}(\chi_2), \gamma_{M_2}(\chi_2))$	$\cdots$	$(\phi_{M_2}(\chi_g), \varphi_{M_2}(\chi_g), \gamma_{M_2}(\chi_g))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
λ_f	$(\phi_{M_f}(\chi_1), \varphi_{M_f}(\chi_1), \gamma_{M_f}(\chi_1))$	$(\phi_{M_f}(\chi_2), \varphi_{M_f}(\chi_2), \gamma_{M_f}(\chi_2))$	$\cdots$	$(\phi_{M_f}(\chi_g), \varphi_{M_f}(\chi_g), \gamma_{M_f}(\chi_g))$

Step 6. Not all criteria might have equal significance. Let $w^{(t)}_b = \left(\phi^{(t)}_w(\chi_b), \varphi^{(t)}_w(\chi_b), \gamma^{(t)}_w(\chi_b)\right)$ represent the PNN that the expert ε_t assigned for the relative weight of criterion χ_b. By aggregating the opinions of experts on χ_b, determine the PN weight $w_b = (\phi_w(\chi_b), \varphi_w(\chi_b), \gamma_w(\chi_b))$ as follows:

$$\begin{aligned} w_b &= PNWA_\ell\left(w^{(1)}_b, w^{(2)}_b, w^{(3)}_b, \ldots, w^{(h)}_b\right)\\ &= \ell_1 w^{(1)}_b \oplus \ell_2 w^{(2)}_b \oplus \ell_3 w^{(3)}_b \oplus \cdots \oplus \ell_h w^{(h)}_b\\ &= \left(\sqrt{1-\prod_{t=1}^{h}\left(1-\left(\phi^{(t)}_b\right)^2\right)^{\ell_t}}, \prod_{t=1}^{h}\left(\varphi^{(t)}_b\right)^{\ell_t}, \prod_{t=1}^{h}\left(\gamma^{(t)}_b\right)^{\ell_t}\right)\end{aligned} \tag{16}$$

Thus, the following criteria weight row matrix can be obtained:

$$w_b = \begin{pmatrix} (\phi_w(\chi_1), \varphi_w(\chi_1), \gamma_w(\chi_1)) \\ (\phi_w(\chi_2), \varphi_w(\chi_2), \gamma_w(\chi_2)) \\ \vdots \\ (\phi_w(\chi_g), \varphi_w(\chi_g), \gamma_w(\chi_g)) \end{pmatrix}^T \tag{17}$$

It is established the weight matrix $w = \{w_1, w_2, w_3, \ldots, w_b, \ldots, w_g\}$ for the criteria. The following equation is used to determine the normalized weights of each criterion based on the total weights of the criteria w_b, which adhere to the condition $\sum_{b=1}^{g} w'_b = 1$.

$$w'_b = \frac{\phi_w(\chi_b) + \gamma_w(\chi_b)\left(\frac{\phi_w(\chi_b)}{\phi_w(\chi_b)+\varphi_w(\chi_b)}\right)}{\sum_{b=1}^{g}\left(\phi_w(\chi_b) + \gamma_w(\chi_b)\left(\frac{\phi_w(\chi_b)}{\phi_w(\chi_b)+\varphi_w(\chi_b)}\right)\right)} \tag{18}$$

Step 7. The weighted normalized aggregated PNDM, $\hat{M} = \left(\hat{M}_{ab}\right)_{f\times g}$, is created, as shown in Matrix 4, in the order of integrating the data from the normalized aggregated PNDM and the criteria weight matrix. The entry $\hat{M}_{ab} = \left(\phi_{\hat{M}_a}(\chi_b), \varphi_{\hat{M}_a}(\chi_b), \gamma_{\hat{M}_a}(\chi_b)\right)$ may be generated by using the specified multiplication operator [3]:

$$\hat{M}_{ab} = (\phi_{M_a}(\chi_b), \varphi_{M_a}(\chi_b), \gamma_{M_a}(\chi_b)) \otimes (\phi_w(\chi_b), \varphi_w(\chi_b), \gamma_w(\chi_b))$$

$$= \left(\phi_{M_a}(\chi_b)\cdot\phi_w(\chi_b),\ \sqrt{\varphi^2_{M_a}(\chi_b) + \varphi^2_w(\chi_b) - \varphi^2_{M_a}(\chi_b)\cdot\varphi^2_w(\chi_b)},\ \sqrt{\gamma^2_{M_a}(\chi_b) + \gamma^2_w(\chi_b) - \gamma^2_{M_a}(\chi_b)\cdot\gamma^2_w(\chi_b)}\right) \tag{19}$$

Matrix 4: Weighted Normalized Aggregated PNDM

$\hat{M}$	χ_1	χ_2	$\cdots$	χ_g
λ_1	$(\phi_{\hat{M}_1}(\chi_1), \varphi_{\hat{M}_1}(\chi_1), \gamma_{\hat{M}_1}(\chi_1))$	$(\phi_{\hat{M}_1}(\chi_2), \varphi_{\hat{M}_1}(\chi_2), \gamma_{\hat{M}_1}(\chi_2))$	$\cdots$	$(\phi_{\hat{M}_1}(\chi_g), \varphi_{\hat{M}_1}(\chi_g), \gamma_{\hat{M}_1}(\chi_g))$
λ_2	$(\phi_{\hat{M}_2}(\chi_1), \varphi_{\hat{M}_2}(\chi_1), \gamma_{\hat{M}_2}(\chi_1))$	$(\phi_{\hat{M}_2}(\chi_2), \varphi_{\hat{M}_2}(\chi_2), \gamma_{\hat{M}_2}(\chi_2))$	$\cdots$	$(\phi_{\hat{M}_2}(\chi_g), \varphi_{\hat{M}_2}(\chi_g), \gamma_{\hat{M}_2}(\chi_g))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
λ_f	$\left(\phi_{\hat{M}_f}(\chi_1), \varphi_{\hat{M}_f}(\chi_1), \gamma_{\hat{M}_f}(\chi_1)\right)$	$\left(\phi_{\hat{M}_f}(\chi_2), \varphi_{\hat{M}_f}(\chi_2), \gamma_{\hat{M}_f}(\chi_2)\right)$	$\cdots$	$\left(\phi_{\hat{M}_f}(\chi_g), \varphi_{\hat{M}_f}(\chi_g), \gamma_{\hat{M}_f}(\chi_g)\right)$

Step 8: De-neutrosophication of weighted normalized aggregated PNDM using preliminaries score value formulas.

Phase II: Pythagorean Neutrosophic Ranking Phase (PN-ELECTRE-III)

The first phase of the Pythagorean decision support system collects the PN assessment data for each alternative, and the second step uses the PN-ELECTRE-III approach, which uses the aggregated evaluations to produce the whole ranking of alternatives. The following steps comes under Module I: Developing Outranking Relations:

Step 9. Establishment of threshold functions. For each criterion χ_b, determine the preference threshold values $p(\chi_b)$ and indifference threshold values $q(\chi_b)$ as shown in paragraph erection of threshold function under Module I of subheading ELECTRE-III method.

Step 10: Calculation of differences in the crisp values which are necessary for applying the cases conditions of partial concordance indices.

Step 11. The partial concordance indices [43,44] are defined as $\Delta_b = [\Delta_b(\lambda_i, \lambda_j)]_{f\times f}$, $(i, j = 1, 2, 3, \ldots, f;\ i \neq j),\ (b = 1, 2, 3, \ldots, g)$, and for each criterion $\chi_b \in X$, these can be obtained using Equation (7), and are illustrated in Matrix 5.

Matrix 5: Partial Concordance Indices Over Each Criterion

Δ_b	λ_1	λ_2	$\cdots$	λ_{f-1}	λ_f
λ_1	$-$	$\Delta_b(\lambda_1, \lambda_2)$	$\cdots$	$\Delta_b(\lambda_1, \lambda_{f-1})$	$\Delta_b(\lambda_1, \lambda_f)$
λ_2	$\Delta_b(\lambda_2, \lambda_1)$	$-$	$\cdots$	$\Delta_b(\lambda_2, \lambda_{f-1})$	$\Delta_b(\lambda_2, \lambda_f)$
$\vdots$	$\vdots$	$\vdots$	$-$	$\vdots$	$\vdots$
λ_{f-1}	$\Delta_b(\lambda_{f-1}, \lambda_1)$	$\Delta_b(\lambda_{f-1}, \lambda_2)$	$\cdots$	$-$	$\Delta_b(\lambda_{f-1}, \lambda_f)$
λ_f	$\Delta_b(\lambda_f, \lambda_1)$	$\Delta_b(\lambda_f, \lambda_2)$	$\cdots$	$\Delta_b(\lambda_f, \lambda_{f-1})$	$-$

And after that, for each pair of alternatives, the comprehensive concordance index

$\Delta = [\Delta_{ij}]_{f\times f},\ (i,j = 1,2,3,\ldots,f;\ i \neq j)$ is calculated using Equation (6), as shown in Matrix 6.

Matrix 6: Comparative Concordance Index

Δ	λ_1	λ_2	$\cdots$	λ_{f-1}	λ_f
λ_1	$-$	Δ_{12}	$\cdots$	$\Delta_{1(f-1)}$	Δ_{1f}
λ_2	Δ_{21}	$-$	$\cdots$	$\Delta_{2(f-1)}$	Δ_{2f}
$\vdots$	$\vdots$	$\vdots$	$-$	$\vdots$	$\vdots$
λ_{f-1}	$\Delta_{(f-1)1}$	$\Delta_{(f-1)2}$	$\cdots$	$-$	$\Delta_{(f-1)f}$
λ_f	Δ_{f1}	Δ_{f2}	$\cdots$	$\Delta_{f(f-1)}$	$-$

Step 12. For each criterion, the discordance index $\nabla_b = [\nabla_b(\lambda_i,\lambda_j)]_{f\times f},\ (i,j = 1,2,3,\ldots,f;\ i \neq j)$ and $(b = 1,2,3,\ldots,g)$ is calculated using Equation (8), as shown in Matrix 7.

Matrix 7: Discordance Index Over Each Criterion

∇_b	λ_1	λ_2	$\cdots$	λ_{f-1}	λ_f
λ_1	$-$	$\nabla_b(\lambda_1,\lambda_2)$	$\cdots$	$\nabla_b(\lambda_1,\lambda_{f-1})$	$\nabla_b(\lambda_1,\lambda_f)$
λ_2	$\nabla_b(\lambda_2,\lambda_1)$	$-$	$\cdots$	$\nabla_b(\lambda_2,\lambda_{f-1})$	$\nabla_b(\lambda_2,\lambda_f)$
$\vdots$	$\vdots$	$\vdots$	$-$	$\vdots$	$\vdots$
λ_{f-1}	$\nabla_b(\lambda_{f-1},\lambda_1)$	$\nabla_b(\lambda_{f-1},\lambda_2)$	$\cdots$	$-$	$\nabla_b(\lambda_{f-1},\lambda_f)$
λ_f	$\nabla_b(\lambda_f,\lambda_1)$	$\nabla_b(\lambda_f,\lambda_2)$	$\cdots$	$\nabla_b(\lambda_f,\lambda_{f-1})$	$-$

Step 13. The credibility index, denoted by $\pi = [\pi_{ij}]_{f\times f},\ (i,j = 1,2,3,\ldots,f;\ i \neq j)$, is used to determine the degree of outranking relation $\lambda_i S \lambda_j$, and can be calculated using Equation (9), as shown in Matrix 8.

Matrix 8: Credibility Index

π	λ_1	λ_2	$\cdots$	λ_{f-1}	λ_f
λ_1	$-$	π_{12}	$\cdots$	$\pi_{1(f-1)}$	π_{1f}
λ_2	π_{21}	$-$	$\cdots$	$\pi_{2(f-1)}$	π_{2f}
$\vdots$	$\vdots$	$\vdots$	$-$	$\vdots$	$\vdots$
λ_{f-1}	$\pi_{(f-1)1}$	$\pi_{(f-1)2}$	$\cdots$	$-$	$\pi_{(f-1)f}$
λ_f	π_{f1}	π_{f2}	$\cdots$	$\pi_{f(f-1)}$	$-$

Module II: The Exploitation of Outranking Relations - To get the comprehensive preference ranking of alternatives, compute the concordance credibility degree, discordance credibility degree, and net credibility degree as described in paragraph Module II under subheading ELECTRE-III method.

Step 14: Outranking of alternatives using credibility index.

Algorithm diagram

The following Figure *1* presents a diagrammatic picture of the two-phase group decision-making support system in order to provide a step-by-step process for problem-solving.

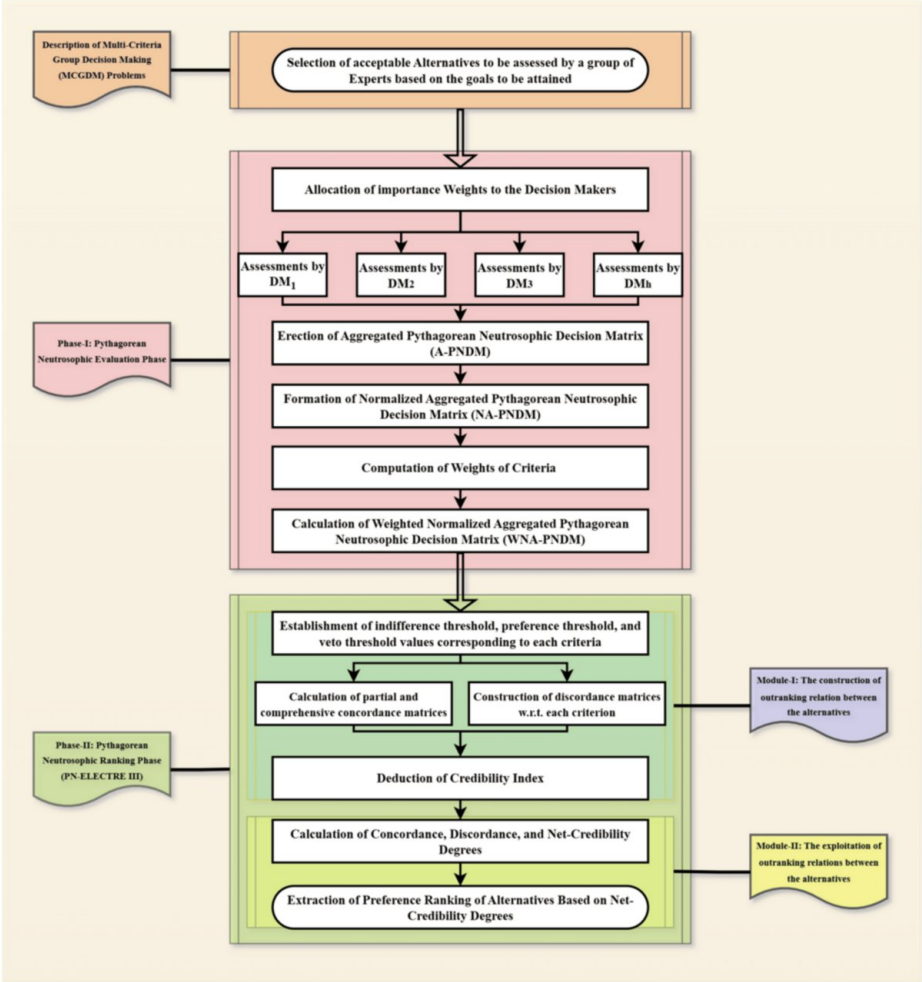
Description of Multi-Criteria Group Decision Making (MCGDM) Problems
Selection of acceptable Alternatives to be assessed by a group of Experts based on the goals to be attained
Allocation of importance Weights to the Decision Makers
Assessments by DM1
Assessments by DM2
Assessments by DM3
Assessments by DMh
Phase-I: Pythagorean Neutrosophic Evaluation Phase
Erection of Aggregated Pythagorean Neutrosophic Decision Matrix (A-PNDM)
Formation of Normalized Aggregated Pythagorean Neutrosophic Decision Matrix (NA-PNDM)
Computation of Weights of Criteria
Calculation of Weighted Normalized Aggregated Pythagorean Neutrosophic Decision Matrix (WNA-PNDM)
Establishment of indifference threshold, preference threshold, and veto threshold values corresponding to each criteria
Calculation of partial and comprehensive concordance matrices
Construction of discordance matrices w.r.t. each criterion
Module-I: The construction of outranking relation between the alternatives
Phase-II: Pythagorean Neutrosophic Ranking Phase (PN-ELECTRE III)
Deduction of Credibility Index
Calculation of Concordance, Discordance, and Net-Credibility Degrees
Module-II: The exploitation of outranking relations between the alternatives
Extraction of Preference Ranking of Alternatives Based on Net-Credibility Degrees

FIGURE 1: Two-Phase Group Decision-Making Support System for Step-by-Step Problem-Solving

Results And Discussion

A case study: solar power plant location selection problem

Bundelkhand is a region in central India, covering parts of Uttar Pradesh and Madhya Pradesh. The Bundelkhand region has a high potential for solar power generation due to its abundant sunlight and vast areas of flat land. According to the India Meteorological Department (IMD), the Bundelkhand region falls under the "Hot and Dry" climate zone, which is characterized by high temperatures and low humidity. The region receives an average of 300-325 days of sunshine per year, making it an ideal location for solar power generation. In recent years, the government of India has also taken various initiatives to promote solar power generation in the region. The region has been identified as a potential hotspot for solar power development under the National Solar Mission.

Uttar Pradesh has a total installed solar capacity of around 29.858 GW, of which the Bundelkhand region's contribution is included. However, it is worth noting that the Uttar Pradesh New and Renewable Energy Development Agency (UPNEDA) has been actively promoting solar power projects in the region. In 2018, UPNEDA invited bids for the development of a 500 MW solar park in Bundelkhand, which would have been one of the largest solar parks in the country. Furthermore, in 2019, UPNEDA invited bids for the development of 1,000 MW of solar power projects across the state, including in the Bundelkhand region.

This section identifies a case study that focuses on solving an MCGDM problem in order to highlight the applicability of the suggested technique in realistic decision-making situations. This study is conducted by one of the Indian NGOs working for solar energy in the Bundelkhand region in Uttar Pradesh, India. The organization is involved in various activities such as promoting renewable energy policies, conducting research on renewable energy technologies, providing training and awareness programs, implementing renewable energy projects, and developing solar power park/plant projects in seven districts of the Bundelkhand region.

Available Alternatives

In this study, Figure 2 shows the map position of the districts that fall under the Bundelkhand region in Uttar Pradesh, which are taken as alternatives: $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7\}$.

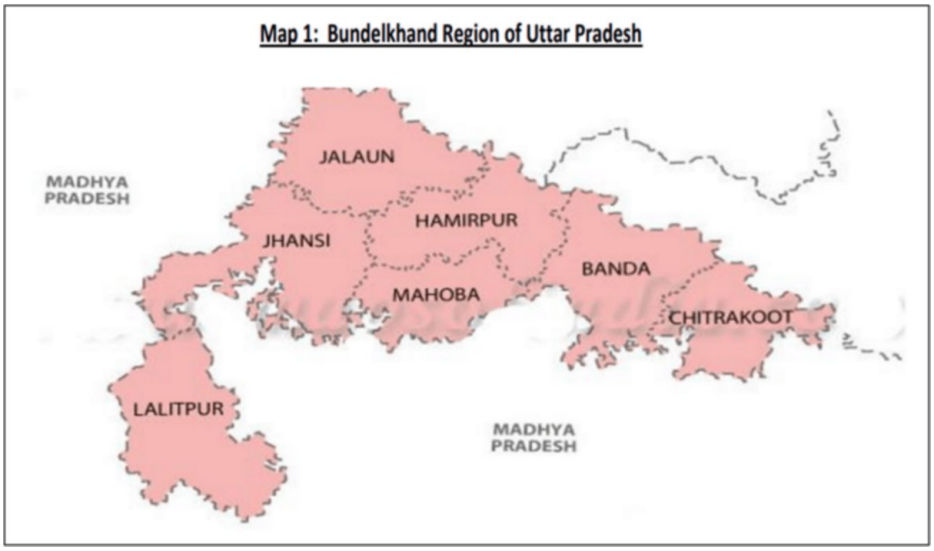

FIGURE 2: Bundelkhand Region of Uttar Pradesh

Map of all seven districts of Bundelkhand region, Uttar Pradesh.

Source: [45]

The following Table 1 shows the alternative cities name with their corresponding codes.

S. No.	Alternative City Name	Alternative Code
1	Banda	λ_1
2	Chitrakoot	λ_2
3	Hamirpur	λ_3
4	Jalaun	λ_4
5	Jhansi	λ_5
6	Lalitpur	λ_6
7	Mahoba	λ_7

TABLE 1: Available Locations (Alternatives) in Seven Districts of Bundelkhand Region

Selection of Criteria

One of the important issues in MCDM analysis is criteria selection. The main criteria that affect solar power plant location selection are economic, geographic and environmental, technical, and social. Choosing the right criteria is crucial to analyze and get accurate results. There are several favourable (Benefit-type/Positive: +) and unfavourable (Cost-type/Negative: -) criteria that can make a location suitable or unsuitable for a solar power plant. Here are some of the top criteria:
$X = \{\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6, \chi_7, \chi_8\}$.

Abundant Solar Radiation (+): The first and foremost factor is the availability of solar radiation. Solar power plants require a significant amount of sunlight to generate electricity. Locations with high levels of annual solar radiation or high numbers of annual sunshine hours are ideal for solar power plants.

Flat and Open Land (+): Solar power plants require large, open spaces to accommodate solar panels. Flat land is ideal as it requires less grading and construction work, reducing the cost of building the solar power plant.

High Land and Construction Costs (-): Solar power plants require large, flat spaces that are expensive to develop. Locations with high land and construction costs may not be financially viable for solar power plants.

Demand for Electricity (+): The population should have a sufficient demand for electricity to justify the installation of a solar power plant. Areas with low population density or low electricity consumption may not be suitable for large-scale solar power plants.

Extreme Weather Conditions (-): Extreme weather conditions such as hurricanes, tornadoes, or heavy snowfall can damage solar panels or disrupt the generation of electricity. Locations with high levels of extreme weather conditions may not be ideal for solar power plants.

Higher Elevation From Sea Level (+): At higher elevations, the air is thinner, which can lead to lower air density and less atmospheric absorption of solar radiation. This can result in higher solar irradiance values and more direct sunlight reaching the solar panels. Additionally, the temperature at higher elevations can be lower, which can help to reduce the operating temperature of the solar panels and increase their efficiency.

Proximity to Transmission Lines (+): Solar power plants generate electricity that needs to be transmitted to the grid. Locations near transmission lines reduce the cost of connecting the solar power plant to the grid.

Average Dust Density (-): Dust density is a measure of the amount of dust particles in the air, and it can have an impact on solar panel efficiency. The presence of dust particles on the surface of the solar panels can reduce the amount of sunlight that reaches the cells, thereby reducing their efficiency.

Table 2 shows the criterion name with its corresponding codes, criteria type, and its source.

S. No.	Criteria Name	Unit	Criteria Code	Type	Source
1	Abundant Solar Radiation	W/m^2	χ_1	+ve	[46]
2	Flat, Open Land	m^2	χ_2	+ve	[46]
3	High Land and Construction Cost	Cost/m^2	χ_3	-ve	[47]
4	Demand for Electricity	kW/Unit	χ_4	+ve	[48]
5	Extreme Weather Conditions	–	χ_5	-ve	[46]
6	Higher Elevation from Sea Level	km	χ_6	+ve	[49]
7	Proximity to Transmission Lines	km	χ_7	+ve	[48]
8	Average Dust Density	mg/m^3	χ_8	-ve	[50]

TABLE 2: List of Most Effective Criteria

Stepwise Procedure

The whole PN-ELECTRE III process is used in the phases that follow to find the best location for the installation of a solar power plant/park unit. The following steps come under the Phase I: Pythagorean Neutrosophic Evaluation Phase:

Step 1: Setting up linguistic terms/variables. By using the decision support system of the suggested technique to solve the aforementioned problem, importance of weight degree to eight criteria and four experts are allocated in the form of linguistic terms/variable that are specified by PNNs as in Table 3.

Linguistic Terms	Code	PNNs
Very High	VH	$(0.92, 0.25, 0.11)$
High	H	$(0.78, 0.35, 0.24)$
Fairly High	FH	$(0.64, 0.42, 0.37)$
Medium	M	$(0.50, 0.55, 0.50)$
Fairly Low	FL	$(0.36, 0.74, 0.63)$
Low	L	$(0.22, 0.85, 0.76)$
Very Low	VL	$(0.08, 0.90, 0.89)$

TABLE 3: Linguistic Terms for Importance Weights Rating of Criteria and Experts

PNNs, Pythagorean Neutrosophic Numbers

In Table 4, specialists independently assess each location's feasibility/performance and threshold functions based on eight criteria, and the performance scores are presented using linguistic terms/variables shown.

Linguistic Terms	Code	PNNs
Extremely Feasible	EF	$(0.99, 0.05, 0.01)$
Very Very Feasible	VVF	$(0.95, 0.35, 0.15)$
Very Feasible	VF	$(0.90, 0.42, 0.25)$
Feasible	F	$(0.80, 0.55, 0.35)$
Medium Feasible	MF	$(0.70, 0.74, 0.40)$
Medium	M	$(0.55, 0.85, 0.45)$
Medium Unfeasible	MU	$(0.45, 0.90, 0.55)$
Unfeasible	U	$(0.40, 0.55, 0.70)$
Very Unfeasible	VU	$(0.35, 0.65, 0.80)$
Very Very Unfeasible	VVU	$(0.25, 0.45, 0.90)$
Extremely Unfeasible	EU	$(0.15, 0.35, 0.95)$

TABLE 4: Linguistic Terms for Feasibility Rating of Alternative and Threshold Functions

PNNs, Pythagorean Neutrosophic Numbers

Step 2: Computing of the weights of decision makers. Table 5 lists the importance rankings that the president of NGO granted to each of the field specialists/decision makers $\varepsilon_t\,(t = 1, 2, 3, \ldots, h)$. Employing Equation (13), it is possible to determine each expert's own weight.

Experts/DMs	Linguistic Variables	PNNs	Crisp Weights (ℓ_t)
ε_1	FH	$(0.64, 0.42, 0.37)$	0.2430
ε_2	VH	$(0.92, 0.25, 0.11)$	0.2832
ε_3	H	$(0.78, 0.35, 0.24)$	0.2832
ε_4	M	$(0.50, 0.55, 0.50)$	0.2077

TABLE 5: Assigning and Computing of Weights of Expert (DM)

DMs, Decision Makers; PNNs, Pythagorean Neutrosophic Numbers

Step 3: Assigning of the decision maker's judgements. The language expressions/linguistic terms used in Matrices 9A-9D describe the individual viewpoints of each decision maker on the decision-making panel with regard to each alternative and all taken-into-account criteria.

Matrices 9A, 9B: Judgments of the First and Second Decision Experts in Linguistic Variables

		χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
ε_1	λ_1	MU	VVF	MU	VF	U	F	MF	MF
	λ_2	VVU	EU	MU	M	EU	U	MF	F
	λ_3	VU	M	VF	MF	MU	VF	EU	VF
	λ_4	MF	M	F	EF	VVF	U	MU	F
	λ_5	M	VF	VVF	MU	VU	EU	M	MU
	λ_6	F	MF	M	MU	VF	EF	U	VU
	λ_7	VVU	MU	MU	MU	VVF	F	F	F

		χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
ε_2	λ_1	VF	M	MU	U	VU	M	VU	M
	λ_2	F	VF	VVF	VF	F	VF	VF	F
	λ_3	M	MU	MU	U	U	MF	MF	VU
	λ_4	F	VF	VVF	VVF	EF	EF	VF	M
	λ_5	M	MF	MU	MU	M	M	U	MU
	λ_6	VF	VF	F	MU	M	MU	MF	VU
	λ_7	U	MF	M	MU	MU	F	F	F

Matrices 9C, 9D: Judgments of the Third and Fourth Decision Experts in Linguistic Variables

		χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
ε_3	λ_1	MU	U	U	VU	VU	VVU	EU	VU
	λ_2	MU	M	MU	EU	VVU	M	MU	MF
	λ_3	F	F	MF	M	MF	MU	VU	VU
	λ_4	EF	VF	VVF	MF	MF	M	M	M
	λ_5	MU	U	MF	VU	VVU	EU	F	F
	λ_6	VVF	F	MF	M	MU	U	U	U
	λ_7	VU	VU	U	M	MU	MU	U	U

		χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
ε_4	λ_1	MF	MU	U	VU	VVU	VU	VU	F
	λ_2	M	MF	MU	U	VU	MF	U	U
	λ_3	MF	M	MU	VU	VVU	EU	U	VF
	λ_4	VVF	EF	VF	VF	F	EF	MF	MF
	λ_5	M	MU	MU	M	VU	VU	VVU	EU
	λ_6	M	MF	MF	F	F	U	VU	VVU
	λ_7	VVU	VU	U	MF	M	U	MF	VF

The PNDM $M^{(1)}, M^{(2)}, M^{(3)}$, and $M^{(4)}$ that highlight the unique opinions of the decision experts $\varepsilon_1, \varepsilon_2, \varepsilon_3,$ and ε_4 in corresponding PNNs equivalence to assigned linguistic variables are shown in Matrices 10A-10D, respectively.

Matrix 10A: Judgement of First Decision Expert in Corresponding PNNs

$M^{(1)}$	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	(0.45, 0.90, 0.55)	(0.95, 0.35, 0.15)	(0.45, 0.90, 0.55)	(0.90, 0.42, 0.25)	(0.40, 0.55, 0.70)	(0.80, 0.55, 0.35)	(0.70, 0.74, 0.40)	(0.70, 0.74, 0.40)
λ_2	(0.25, 0.45, 0.90)	(0.15, 0.35, 0.95)	(0.45, 0.90, 0.55)	(0.55, 0.85, 0.45)	(0.15, 0.35, 0.95)	(0.40, 0.55, 0.70)	(0.70, 0.74, 0.40)	(0.80, 0.55, 0.35)
λ_3	(0.35, 0.65, 0.80)	(0.55, 0.85, 0.45)	(0.90, 0.42, 0.25)	(0.70, 0.74, 0.40)	(0.45, 0.90, 0.55)	(0.90, 0.42, 0.25)	(0.15, 0.35, 0.95)	(0.90, 0.42, 0.25)
λ_4	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.80, 0.55, 0.35)	(0.99, 0.05, 0.01)	(0.95, 0.35, 0.15)	(0.40, 0.55, 0.70)	(0.45, 0.90, 0.55)	(0.80, 0.55, 0.35)
λ_5	(0.55, 0.85, 0.45)	(0.90, 0.42, 0.25)	(0.95, 0.35, 0.15)	(0.45, 0.90, 0.55)	(0.35, 0.65, 0.80)	(0.15, 0.35, 0.95)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)
λ_6	(0.80, 0.55, 0.35)	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.90, 0.42, 0.25)	(0.99, 0.05, 0.01)	(0.40, 0.55, 0.70)	(0.35, 0.65, 0.80)
λ_7	(0.25, 0.45, 0.90)	(0.45, 0.90, 0.55)	(0.45, 0.90, 0.55)	(0.45, 0.90, 0.55)	(0.95, 0.35, 0.15)	(0.80, 0.55, 0.35)	(0.80, 0.55, 0.35)	(0.80, 0.55, 0.35)

Matrix 10B: Judgement of Second Decision Expert in Corresponding PNNs

$M^{(2)}$	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	(0.90, 0.42, 0.25)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.35, 0.65, 0.80)	(0.55, 0.85, 0.45)	(0.35, 0.65, 0.80)	(0.55, 0.85, 0.45)
λ_2	(0.80, 0.55, 0.35)	(0.90, 0.42, 0.25)	(0.95, 0.35, 0.15)	(0.90, 0.42, 0.25)	(0.80, 0.55, 0.35)	(0.90, 0.42, 0.25)	(0.90, 0.42, 0.25)	(0.80, 0.55, 0.35)
λ_3	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.40, 0.55, 0.70)	(0.70, 0.74, 0.40)	(0.70, 0.74, 0.40)	(0.35, 0.65, 0.80)
λ_4	(0.80, 0.55, 0.35)	(0.90, 0.42, 0.25)	(0.95, 0.35, 0.15)	(0.95, 0.35, 0.15)	(0.99, 0.05, 0.01)	(0.99, 0.05, 0.01)	(0.90, 0.42, 0.25)	(0.55, 0.85, 0.45)
λ_5	(0.55, 0.85, 0.45)	(0.70, 0.74, 0.40)	(0.45, 0.90, 0.55)	(0.45, 0.90, 0.55)	(0.55, 0.85, 0.45)	(0.55, 0.85, 0.45)	(0.40, 0.55, 0.70)	(0.45, 0.90, 0.55)
λ_6	(0.90, 0.42, 0.25)	(0.90, 0.42, 0.25)	(0.80, 0.55, 0.35)	(0.45, 0.90, 0.55)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.70, 0.74, 0.40)	(0.35, 0.65, 0.80)
λ_7	(0.40, 0.55, 0.70)	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.45, 0.90, 0.55)	(0.80, 0.55, 0.35)	(0.80, 0.55, 0.35)	(0.80, 0.55, 0.35)

Matrix 10C: Judgement of Third Decision Expert in Corresponding PNNs

$M^{(3)}$	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.40, 0.55, 0.70)	(0.35, 0.65, 0.80)	(0.35, 0.65, 0.80)	(0.25, 0.45, 0.90)	(0.15, 0.35, 0.95)	(0.35, 0.65, 0.80)
λ_2	(0.45, 0.90, 0.55)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.15, 0.35, 0.95)	(0.25, 0.45, 0.90)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.70, 0.74, 0.40)
λ_3	(0.80, 0.55, 0.35)	(0.80, 0.55, 0.35)	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.70, 0.74, 0.40)	(0.45, 0.90, 0.55)	(0.35, 0.65, 0.80)	(0.35, 0.65, 0.80)
λ_4	(0.99, 0.05, 0.01)	(0.90, 0.42, 0.25)	(0.95, 0.35, 0.15)	(0.70, 0.74, 0.40)	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.55, 0.85, 0.45)	(0.55, 0.85, 0.45)
λ_5	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.70, 0.74, 0.40)	(0.35, 0.65, 0.80)	(0.25, 0.45, 0.90)	(0.15, 0.35, 0.95)	(0.80, 0.55, 0.35)	(0.80, 0.55, 0.35)
λ_6	(0.95, 0.35, 0.15)	(0.80, 0.55, 0.35)	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.40, 0.55, 0.70)	(0.40, 0.55, 0.70)
λ_7	(0.35, 0.65, 0.80)	(0.35, 0.65, 0.80)	(0.40, 0.55, 0.70)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.40, 0.55, 0.70)

Matrix 10D: Judgement of Fourth Decision Expert in Corresponding PNNs

$M^{(4)}$	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	(0.70, 0.74, 0.40)	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.35, 0.65, 0.80)	(0.25, 0.45, 0.90)	(0.35, 0.65, 0.80)	(0.35, 0.65, 0.80)	(0.80, 0.55, 0.35)
λ_2	(0.55, 0.85, 0.45)	(0.70, 0.74, 0.40)	(0.45, 0.90, 0.55)	(0.40, 0.55, 0.70)	(0.35, 0.65, 0.80)	(0.70, 0.74, 0.40)	(0.40, 0.55, 0.70)	(0.40, 0.55, 0.70)
λ_3	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.35, 0.65, 0.80)	(0.25, 0.45, 0.90)	(0.15, 0.35, 0.95)	(0.40, 0.55, 0.70)	(0.90, 0.42, 0.25)
λ_4	(0.95, 0.35, 0.15)	(0.99, 0.05, 0.01)	(0.90, 0.42, 0.25)	(0.90, 0.42, 0.25)	(0.80, 0.55, 0.35)	(0.99, 0.05, 0.01)	(0.70, 0.74, 0.40)	(0.70, 0.74, 0.40)
λ_5	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.45, 0.90, 0.55)	(0.55, 0.85, 0.45)	(0.35, 0.65, 0.80)	(0.35, 0.65, 0.80)	(0.25, 0.45, 0.90)	(0.15, 0.35, 0.95)
λ_6	(0.55, 0.85, 0.45)	(0.70, 0.74, 0.40)	(0.70, 0.74, 0.40)	(0.80, 0.55, 0.35)	(0.80, 0.55, 0.35)	(0.40, 0.55, 0.70)	(0.35, 0.65, 0.80)	(0.25, 0.45, 0.90)
λ_7	(0.25, 0.45, 0.90)	(0.35, 0.65, 0.80)	(0.40, 0.55, 0.70)	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)	(0.40, 0.55, 0.70)	(0.70, 0.74, 0.40)	(0.90, 0.42, 0.25)

Step 4: Aggregation of the decision makers' judgements. According to the PNWA operator and the normalized weights assigned to each decision-making expert, the individual assessments provided by each expert are combined. This process results in the construction of an aggregated PNDM using Equation (14), denoted by $\ddot{M} = \left(\ddot{M}_{ab}\right)_{7\times 8}$, where $\ddot{M}_{ab}$ represents the aggregated PNN corresponding to alternative Λ_a under criterion χ_b, for $a = 1, 2, \ldots, 7$ and $b = 1, 2, \ldots, 8$. Matrix 11 presents this aggregated decision matrix.

Matrix 11: Aggregated PNDM

M	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	(0.7182, 0.6964, 0.4118)	(0.7300, 0.6175, 0.4040)	(0.4274, 0.7127, 0.6166)	(0.6344, 0.5575, 0.5807)	(0.3461, 0.5782, 0.7936)	(0.5710, 0.6106, 0.5737)	(0.4566, 0.5689, 0.7076)	(0.6333, 0.6991, 0.4837)
λ_2	(0.5967, 0.6537, 0.5232)	(0.7135, 0.5452, 0.4458)	(0.7485, 0.6888, 0.3807)	(0.6718, 0.5022, 0.5095)	(0.5363, 0.4837, 0.6810)	(0.7259, 0.6085, 0.4139)	(0.7199, 0.6243, 0.4281)	(0.7274, 0.5952, 0.4188)
λ_3	(0.6484, 0.6891, 0.4724)	(0.6265, 0.7694, 0.4455)	(0.7073, 0.7099, 0.4172)	(0.5342, 0.6872, 0.5585)	(0.5069, 0.6435, 0.5993)	(0.6948, 0.5815, 0.4648)	(0.4842, 0.5604, 0.6667)	(0.7481, 0.5339, 0.4736)
λ_4	(0.9283, 0.2843, 0.1177)	(0.9148, 0.3204, 0.1478)	(0.9199, 0.4057, 0.2049)	(0.9390, 0.2765, 0.1121)	(0.9345, 0.2704, 0.1078)	(0.9342, 0.1903, 0.0773)	(0.7299, 0.6859, 0.3904)	(0.6659, 0.7430, 0.4131)
λ_5	(0.5265, 0.8630, 0.4747)	(0.7052, 0.6206, 0.4424)	(0.7584, 0.6791, 0.3685)	(0.4521, 0.8156, 0.5829)	(0.4039, 0.6359, 0.7014)	(0.3623, 0.5117, 0.7419)	(0.5868, 0.5864, 0.5509)	(0.5716, 0.6488, 0.5463)
λ_6	(0.8698, 0.4946, 0.2676)	(0.8053, 0.5825, 0.3379)	(0.7081, 0.7037, 0.3963)	(0.5896, 0.8002, 0.4747)	(0.7353, 0.6643, 0.3906)	(0.8165, 0.3531, 0.2329)	(0.5139, 0.6193, 0.6142)	(0.3473, 0.5760, 0.7912)
λ_7	(0.3272, 0.5253, 0.8123)	(0.5146, 0.7298, 0.6002)	(0.4616, 0.7013, 0.5825)	(0.5465, 0.8511, 0.4880)	(0.7311, 0.7070, 0.3847)	(0.6852, 0.6270, 0.4559)	(0.7177, 0.5850, 0.4327)	(0.7778, 0.5200, 0.3925)

Step 5: Normalization of aggregated PNDM. Let X_B and X_C represent the sets of criteria that are benefit-type (positive) and cost-type (negative), respectively. That is, $\chi_B = \{\chi_1, \chi_2, \chi_4, \chi_6, \chi_7\}$, $\chi_C = \{\chi_3, \chi_5, \chi_8\}$. Using Equation (15), the aggregated PNDM, denoted by $\ddot{M} = \left(\ddot{M}_{ab}\right)_{7\times 8}$, is transformed into the normalized aggregated PNDM, $M = (M_{ab})_{7\times 8}$, which standardizes the evaluation of each alternative with respect to every criterion in a uniform format (considering whether the criterion is beneficial or costly), and facilitates further analysis. Matrix 12 illustrates the construction of the normalized APNDM *M*, where each element M_{ab} is expressed as a PNN.

Matrix 12: Normalized Aggregated PNDM

M	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	(0.7182, 0.6964, 0.4118)	(0.7300, 0.6175, 0.4040)	(0.6166, 0.7127, 0.4274)	(0.6344, 0.5575, 0.5807)	(0.7936, 0.5782, 0.3461)	(0.5710, 0.6106, 0.5737)	(0.4566, 0.5689, 0.7076)	(0.4837, 0.6991, 0.6333)
λ_2	(0.5967, 0.6537, 0.5232)	(0.7135, 0.5452, 0.4458)	(0.3807, 0.6888, 0.7485)	(0.6718, 0.5022, 0.5095)	(0.6810, 0.4837, 0.5363)	(0.7259, 0.6085, 0.4139)	(0.7199, 0.6243, 0.4281)	(0.4188, 0.5952, 0.7274)
λ_3	(0.6484, 0.6891, 0.4724)	(0.6265, 0.7694, 0.4455)	(0.4172, 0.7099, 0.7073)	(0.5342, 0.6872, 0.5585)	(0.5993, 0.6435, 0.5069)	(0.6948, 0.5815, 0.4648)	(0.4842, 0.5604, 0.6667)	(0.4736, 0.5339, 0.7481)
λ_4	(0.9283, 0.2843, 0.1177)	(0.9148, 0.3204, 0.1478)	(0.2049, 0.4057, 0.9199)	(0.9390, 0.2765, 0.1121)	(0.1078, 0.2704, 0.9345)	(0.9342, 0.1903, 0.0773)	(0.7299, 0.6859, 0.3904)	(0.4131, 0.7430, 0.6659)
λ_5	(0.5265, 0.8630, 0.4747)	(0.7052, 0.6206, 0.4424)	(0.3685, 0.6791, 0.7584)	(0.4521, 0.8156, 0.5829)	(0.7014, 0.6359, 0.4039)	(0.3623, 0.5117, 0.7419)	(0.5868, 0.5864, 0.5509)	(0.5463, 0.6488, 0.5716)
λ_6	(0.8698, 0.4946, 0.2676)	(0.8053, 0.5825, 0.3379)	(0.3963, 0.7037, 0.7081)	(0.5896, 0.8002, 0.4747)	(0.3906, 0.6643, 0.7353)	(0.8165, 0.3531, 0.2329)	(0.5139, 0.6193, 0.6142)	(0.7912, 0.5760, 0.3473)
λ_7	(0.3272, 0.5253, 0.8123)	(0.5146, 0.7298, 0.6002)	(0.5825, 0.7013, 0.4616)	(0.5465, 0.8511, 0.4880)	(0.3847, 0.7070, 0.7311)	(0.6852, 0.6270, 0.4559)	(0.7177, 0.5850, 0.4327)	(0.3925, 0.5200, 0.7778)

Step 6: Erection of weight matrix of criteria. The decision-making panel's linguistic labels for each criterion. PN-weights in linguistic variable are shown in Matrix 13A. Matrix 13B shows the criteria weights in the corresponding PNNs.

Matrix 13A: Linguistic Variables to Unfold Importance of Criteria

W_ε^χ	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
ε_1	VH	VH	FH	M	VL	FL	M	FH
ε_2	H	VH	VH	M	VL	FL	H	H
ε_3	M	H	FH	VH	BL	M	BL	M
ε_4	FH	VH	FL	VL	VL	H	VH	M

Matrix 13B: PNNs to Unfold Importance of Criteria

W_ε^χ	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
ε_1	(0.92, 0.25, 0.11)	(0.92, 0.25, 0.11)	(0.84, 0.2, 0.27)	(0.62, 0.35, 0.34)	(0.26, 0.72, 0.65)	(0.46, 0.52, 0.49)	(0.61, 0.42, 0.38)	(0.84, 0.28, 0.26)
ε_2	(0.81, 0.35, 0.34)	(0.92, 0.25, 0.11)	(0.92, 0.25, 0.11)	(0.62, 0.35, 0.34)	(0.26, 0.72, 0.65)	(0.46, 0.52, 0.49)	(0.81, 0.35, 0.34)	(0.81, 0.35, 0.34)
ε_3	(0.61, 0.35, 0.35)	(0.81, 0.35, 0.34)	(0.84, 0.2, 0.27)	(0.92, 0.25, 0.11)	(0.35, 0.63, 0.54)	(0.61, 0.42, 0.38)	(0.35, 0.63, 0.54)	(0.61, 0.35, 0.34)
ε_4	(0.84, 0.2, 0.27)	(0.92, 0.25, 0.11)	(0.72, 0.35, 0.24)	(0.26, 0.72, 0.65)	(0.26, 0.72, 0.65)	(0.81, 0.35, 0.34)	(0.92, 0.25, 0.11)	(0.61, 0.35, 0.34)

Using Equations (16)-(18), the criteria weights vector and corresponding crisp weights of the criteria are given as:

$$W_{\{\chi_1,\chi_2,\chi_3,\chi_4,\chi_5,\chi_6,\chi_7,\chi_8\}} = \begin{pmatrix} (0.7721, 0.3778, 0.2641) \\ (0.8961, 0.2734, 0.1354) \\ (0.5850, 0.5272, 0.4461) \\ (0.7221, 0.4442, 0.3259) \\ (0.6660, 0.4753, 0.3766) \\ (0.4682, 0.6215, 0.5617) \\ (0.7267, 0.4482, 0.3279) \\ (0.4444, 0.6455, 0.5898) \end{pmatrix}^T \quad \text{and} \quad \begin{matrix} w(\chi_1) = 0.1375 \\ w(\chi_2) = 0.1449 \\ w(\chi_3) = 0.1187 \\ w(\chi_4) = 0.1338 \\ w(\chi_5) = 0.1283 \\ w(\chi_6) = 0.1028 \\ w(\chi_7) = 0.1347 \\ w(\chi_8) = 0.0992 \end{matrix}$$

Step 7: Construction of weighted normalized aggregation PNDM. The weighted normalized aggregated PNDM, $\hat{M} = \left(\hat{M}_{ab}\right)_{f\times g}$, is created, as shown in Matrix 14, in the order of integrating the data from the normalized aggregated PNDM and the criteria weight matrix.

$\hat{M}_{ab} = \left(\phi_{\hat{M}_a}(\chi_b),\ \varphi_{\hat{M}_a}(\chi_b),\ \gamma_{\hat{M}_a}(\chi_b)\right)$ may be generated by using the multiplication operator, which is specified in Equation (19).

Matrix 14: Weighted Normalized Aggregated PNDM

$\hat{M}$	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	(0.5545, 0.7473, 0.4770)	(0.6542, 0.6539, 0.4226)	(0.3607, 0.8029, 0.5876)	(0.4581, 0.6684, 0.6384)	(0.5285, 0.6962, 0.4946)	(0.2673, 0.7843, 0.7354)	(0.3318, 0.6779, 0.7446)	(0.2150, 0.8377, 0.7807)
λ_2	(0.4607, 0.7135, 0.5696)	(0.6394, 0.5914, 0.4620)	(0.2227, 0.7877, 0.8048)	(0.4851, 0.6323, 0.5816)	(0.4535, 0.6380, 0.6234)	(0.3399, 0.7833, 0.6579)	(0.5232, 0.7158, 0.5207)	(0.1861, 0.7895, 0.8324)
λ_3	(0.5006, 0.7415, 0.5266)	(0.5614, 0.7890, 0.4617)	(0.2441, 0.8011, 0.7744)	(0.3857, 0.7592, 0.6205)	(0.3991, 0.7392, 0.6019)	(0.3253, 0.7706, 0.6807)	(0.3519, 0.6722, 0.7101)	(0.2105, 0.7635, 0.8443)
λ_4	(0.7167, 0.4605, 0.2875)	(0.8198, 0.4120, 0.1994)	(0.1199, 0.6299, 0.9364)	(0.6781, 0.5086, 0.3427)	(0.0718, 0.5315, 0.9441)	(0.4374, 0.6391, 0.5653)	(0.5304, 0.7595, 0.4935)	(0.1836, 0.8595, 0.7981)
λ_5	(0.4065, 0.8839, 0.5286)	(0.6319, 0.6566, 0.4588)	(0.2156, 0.7816, 0.8122)	(0.3265, 0.8551, 0.6402)	(0.4671, 0.7341, 0.5309)	(0.1696, 0.7396, 0.8320)	(0.4264, 0.6897, 0.6151)	(0.2428, 0.8138, 0.7490)
λ_6	(0.6716, 0.5937, 0.3693)	(0.7216, 0.6235, 0.3611)	(0.2318, 0.7972, 0.7750)	(0.4258, 0.8434, 0.5546)	(0.2601, 0.7533, 0.7783)	(0.3823, 0.6803, 0.5938)	(0.3735, 0.7123, 0.6665)	(0.3516, 0.7812, 0.6531)
λ_7	(0.2526, 0.6159, 0.8268)	(0.4611, 0.7534, 0.6099)	(0.3408, 0.7957, 0.6080)	(0.3946, 0.8825, 0.5649)	(0.2562, 0.7828, 0.7749)	(0.3208, 0.7922, 0.6766)	(0.5216, 0.6887, 0.5240)	(0.1744, 0.7579, 0.8616)

Step 8: Computation of score degrees with respect to WNA-PNDM. Matrix 15 contains the computed crisp values or score values of corresponding PNNs in the weighted normalized aggregated PNDM $\hat{M}$.

Matrix 15: Score Value of Weighted Normalized Aggregated PNDM)

M_{Crisp}	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
λ_1	−0.4785	−0.1782	−0.8598	−0.6445	−0.4500	−1.0845	−0.9039	−1.2650
λ_2	−0.6213	−0.1544	−1.2186	−0.5027	−0.5900	−0.9309	−0.5098	−1.2816
λ_3	−0.5765	−0.5205	−1.1819	−0.8126	−0.7494	−0.9514	−0.8323	−1.2515
λ_4	0.2189	0.4626	−1.2592	0.0837	−1.1687	−0.5367	−0.5391	−1.3420
λ_5	−0.8955	−0.2423	−1.2241	−1.0344	−0.6026	−1.2105	−0.6722	−1.1643
λ_6	−0.0378	0.0016	−1.1824	−0.8376	−1.1056	−0.6693	−0.8121	−0.9132
λ_7	−0.9991	−0.7270	−0.8867	−0.9422	−1.1476	−0.9825	−0.4768	−1.2864

The following steps comes under Phase II: Pythagorean Neutrosophic Ranking Phase (PN-ELECTRE III). Firstly, we will start with Module I, i.e. the construction of outranking relations:

Step 9. Establishment of threshold functions. For each criteria χ_b, here are preference threshold values $p(\chi_b)$ and indifference threshold values $q(\chi_b)$ and veto threshold values $v(\chi_b)$, as shown in Matrix 16 with respect to linguistic variable, corresponding PNNs, and its crisp or score values.

Matrix 16: Assigning of PNN's Threshold Functions and Its Score Values

T_χ	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
$q(\chi_j)$	F	EF	MF	VF	M	U	VVF	MU
	(0.80, 0.55, 0.35)	(0.99, 0.05, 0.01)	(0.70, 0.74, 0.40)	(0.90, 0.42, 0.25)	(0.55, 0.85, 0.45)	(0.40, 0.55, 0.70)	(0.95, 0.35, 0.15)	(0.45, 0.90, 0.55)
$s(q(\chi_j))$	0.2150	0.9775	−0.2176	0.5711	−0.6225	−0.6325	0.7575	−0.9100
$p(\chi_j)$	MU	M	M	F	MF	VU	MF	M
	(0.45, 0.90, 0.55)	(0.55, 0.85, 0.45)	(0.55, 0.85, 0.45)	(0.80, 0.55, 0.35)	(0.70, 0.74, 0.40)	(0.35, 0.65, 0.80)	(0.70, 0.74, 0.40)	(0.55, 0.85, 0.45)
$s(p(\chi_j))$	−0.9100	−0.6225	−0.6225	0.2150	−0.2176	−0.9400	−0.2176	−0.6225
$v(\chi_j)$	EF	MF	VF	M	MU	M	M	MF
	(0.99, 0.05, 0.01)	(0.70, 0.74, 0.40)	(0.90, 0.42, 0.25)	(0.55, 0.85, 0.45)	(0.45, 0.90, 0.55)	(0.55, 0.85, 0.45)	(0.55, 0.85, 0.45)	(0.70, 0.74, 0.40)
$s(v(\chi_j))$	0.9775	−0.2176	0.5711	−0.6225	−0.6700	−0.6225	−0.6225	−0.2176

Step 10. Calculation of difference in the score degrees. This calculation is helpful for the implementation of Equations (6)-(8), which are going to be used in next steps 11 and 12. The differences in the score values/degrees of the feasibility of every pair of alternatives are computing of $\lambda_{ij}^{\chi_b} = s\left(\chi_b(\lambda_j)\right) - s\left(\chi_b(\lambda_i)\right) \quad \forall i, j = 1, 2, 3, ..., 7, i \neq j$, where b = 1, 2, ... 8, in Matrices 17A-17H as follows:

Matrices 17A, 17B: $\lambda_{ij}^{\chi_1}$ and $\lambda_{ij}^{\chi_2} \forall i, j = 1, 2, 3, ..., 7, i \neq j$

$$\lambda_{ij}^{\chi_1} = \begin{matrix} - & -0.1428 & -0.098 & 0.6974 & -0.417 & 0.4407 & -0.5206 \\ 0.1428 & - & 0.0448 & 0.8402 & -0.2742 & 0.5835 & -0.3778 \\ 0.098 & -0.0448 & - & 0.7954 & -0.319 & 0.5387 & -0.4226 \\ -0.6974 & -0.8402 & -0.7954 & - & -1.1144 & -0.2567 & -1.218 \\ 0.417 & 0.2742 & 0.319 & 1.1144 & - & 0.8577 & -0.1036 \\ -0.4407 & -0.5835 & -0.5387 & 0.2567 & -0.8577 & - & -0.9613 \\ 0.5206 & 0.3778 & 0.4226 & 1.218 & 0.1036 & 0.9613 & - \end{matrix} \qquad \lambda_{ij}^{\chi_2} = \begin{matrix} - & 0.0238 & -0.3423 & 0.6408 & -0.0641 & 0.1798 & -0.5488 \\ -0.0238 & - & -0.3661 & 0.617 & -0.0879 & 0.156 & -0.5726 \\ 0.3423 & 0.3661 & - & 0.9831 & 0.2782 & 0.5221 & -0.2065 \\ -0.6408 & -0.617 & -0.9831 & - & -0.7049 & -0.461 & -1.1896 \\ 0.0641 & 0.0879 & -0.2782 & 0.7049 & - & 0.2439 & -0.4847 \\ -0.1798 & -0.156 & -0.5221 & 0.461 & -0.2439 & - & -0.7286 \\ 0.5488 & 0.5726 & 0.2065 & 1.1896 & 0.4847 & 0.7286 & - \end{matrix}$$

Matrices 17C, 17D: $\lambda_{ij}^{\chi_3}$ and $\lambda_{ij}^{\chi_4} \forall i, j = 1, 2, 3, ..., 7, i \neq j$

$\lambda_{ij}^{\chi_3} =$

–	−0.3588	−0.3221	−0.3994	−0.3643	−0.3226	−0.0269
0.3588	–	0.0367	−0.0406	−0.0055	0.0362	0.3319
0.3221	−0.0367	–	−0.0773	−0.0422	−0.0005	0.2952
0.3994	0.0406	0.0773	–	0.0351	0.0768	0.3725
0.3643	0.0055	0.0422	−0.0351	–	0.0417	0.3374
0.3226	−0.0362	0.0005	−0.0768	−0.0417	–	0.2957
0.0269	−0.3319	−0.2952	−0.3725	−0.3374	−0.2957	–

$\lambda_{ij}^{\chi_4} =$

–	0.1418	−0.1681	0.7282	−0.3899	−0.1931	−0.2977
−0.1418	–	−0.3099	0.5864	−0.5317	−0.3349	−0.4395
0.1681	0.3099	–	0.8963	−0.2218	−0.025	−0.1296
−0.7282	−0.5864	−0.8963	–	−1.1181	−0.9213	−1.0259
0.3899	0.5317	0.2218	1.1181	–	0.1968	0.0922
0.1931	0.3349	0.025	0.9213	−0.1968	–	−0.1046
0.2977	0.4395	0.1296	1.0259	−0.0922	0.1046	–

Matrices 17E, 17F: $\lambda_{ij}^{\chi_5}$ and $\lambda_{ij}^{\chi_6} \forall i, j = 1, 2, 3, ..., 7, i \neq j$

$\lambda_{ij}^{\chi_5} =$

–	−0.14	−0.2994	−0.7187	−0.1526	−0.6556	−0.6976
0.14	–	−0.1594	−0.5787	−0.0126	−0.5156	−0.5576
0.2994	0.1594	–	−0.4193	0.1468	−0.3562	−0.3982
0.7187	0.5787	0.4193	–	0.5661	0.0631	0.0211
0.1526	0.0126	−0.1468	−0.5661	–	−0.503	−0.545
0.6556	0.5156	0.3562	−0.0631	0.503	–	−0.042
0.6976	0.5576	0.3982	−0.0211	0.545	0.042	–

$\lambda_{ij}^{\chi_6} =$

–	0.1536	0.1331	0.5478	−0.126	0.4152	0.102
−0.1536	–	−0.0205	0.3942	−0.2796	0.2616	−0.0516
−0.1331	0.0205	–	0.4147	−0.2591	0.2821	−0.0311
−0.5478	−0.3942	−0.4147	–	−0.6738	−0.1326	−0.4458
0.126	0.2796	0.2591	0.6738	–	0.5412	0.228
−0.4152	−0.2616	−0.2821	0.1326	−0.5412	–	−0.3132
−0.102	0.0516	0.0311	0.4458	−0.228	0.3132	–

Matrices 17G, 17H: $\lambda_{ij}^{\chi_7}$ and $\lambda_{ij}^{\chi_8} \forall i, j = 1, 2, 3, ..., 7, i \neq j$

$\lambda_{ij}^{\chi_7} =$

–	0.3941	0.0716	0.3648	0.2317	0.0918	0.4271
−0.3941	–	−0.3225	−0.0293	−0.1624	−0.3023	0.033
−0.0716	0.3225	–	0.2932	0.1601	0.0202	0.3555
−0.3648	0.0293	−0.2932	–	−0.1331	−0.273	0.0623
−0.2317	0.1624	−0.1601	0.1331	–	−0.1399	0.1954
−0.0918	0.3023	−0.0202	0.273	0.1399	–	0.3353
−0.4271	−0.033	−0.3555	−0.0623	−0.1954	−0.3353	–

$\lambda_{ij}^{\chi_8} =$

–	−0.0166	0.0135	−0.077	0.1007	0.3518	−0.0214
0.0166	–	0.0301	−0.0604	0.1173	0.3684	−0.0048
−0.0135	−0.0301	–	−0.0905	0.0872	0.3383	−0.0349
0.077	0.0604	0.0905	–	0.1777	0.4288	0.0556
−0.1007	−0.1173	−0.0872	−0.1777	–	0.2511	−0.1221
−0.3518	−0.3684	−0.3383	−0.4288	−0.2511	–	−0.3732
0.0214	0.0048	0.0349	−0.0556	0.1221	0.3732	–

Step 11: Calculation of partial concordance indices and concordance matrix. The partial concordance matrices shown in Matrices 18A-18H are calculated using Equation (7).

Matrices 18A, 18B: Partial Concordance Indices w.r.t. First and Second Criteria

Δ_1	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0	0	0	0	0
λ_2	0	–	0	0	0	0	0
λ_3	0	0	–	0	0	0	0
λ_4	0.2287	0.6647	0.5279	–	1	0	1
λ_5	0	0	0	0	–	0	0
λ_6	0	0	0	0	0.7182	–	1
λ_7	0	0	0	0	0	0	–

Δ_2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0.1726	0	0	0	0.4585
λ_2	0	–	0.2056	0	0	0	0.4914
λ_3	0	0	–	0	0	0	0
λ_4	0.5858	0.5529	1	–	0.6746	0.3369	1
λ_5	0	0	0.0839	0	–	0	0.3697
λ_6	0	0	0.4215	0	0.0364	–	0.7074
λ_7	0	0	0	0	0	0	–

Matrices 18C, 18D: Partial Concordance Indices w.r.t. Third and Fourth Criteria

Δ_3	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	1	1	1	1	1	0.5592
λ_2	0	–	0.4122	0.5908	0.5097	0.4133	0
λ_3	0	0.5818	–	0.6757	0.5945	0.4982	0
λ_4	0	0.4031	0.3183	–	0.4159	0.3195	0
λ_5	0	0.4843	0.3994	0.5781	–	0.4006	0
λ_6	0	0.5807	0.4958	0.6745	0.5934	–	0
λ_7	0.4348	1	1	1	1	1	–

Δ_4	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0	0	0.2488	0	0.1157
λ_2	0	–	0.1333	0	0.4536	0.1694	0.3205
λ_3	0	0	–	0	0.0061	0	0
λ_4	0.7374	0.5326	0.9802	–	1	1	1
λ_5	0	0	0	0	–	0	0
λ_6	0	0	0	0	0	–	0
λ_7	0	0	0	0	0	0	–

Matrices 18E, 18F: Partial Concordance Indices w.r.t. Fifth and Sixth Criteria

Δ_5	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.9016	1	1	0.9176	1	1
λ_2	0.5466	–	0.9262	1	0.7401	1	1
λ_3	0.3445	0.522	–	1	0.538	1	1
λ_4	0	0	0.1925	–	0.0063	0.6441	0.6974
λ_5	0.5306	0.7081	0.9102	1	–	1	1
λ_6	0	0.0704	0.2725	0.8041	0.0863	–	0.7774
λ_7	0	0.0171	0.2192	0.7509	0.0331	0.6709	–

Δ_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.0733	0.0978	0	0.4072	0	0.1349
λ_2	0.4401	–	0.2812	0	0.5906	0	0.3183
λ_3	0.4156	0.2322	–	0	0.5661	0	0.2939
λ_4	0.9108	0.7274	0.7519	–	1	0.415	0.789
λ_5	0.1063	0	0	0	–	0	0
λ_6	0.7525	0.5691	0.5936	0.0984	0.9029	–	0.6307
λ_7	0.3785	0.1951	0.2196	0	0.529	0	–

Matrices 18G, 18H: Partial Concordance Indices w.r.t. Seventh and Eighth Criteria

Δ_7	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0.1712	0	0	0.1471	0
λ_2	0.7273	–	0.6418	0.2917	0.4506	0.6177	0.2173
λ_3	0.3422	0	–	0	0.0656	0.2326	0
λ_4	0.6923	0.2217	0.6068	–	0.4156	0.5827	0.1823
λ_5	0.5334	0.0628	0.4479	0.0978	–	0.4238	0.0234
λ_6	0.3663	0	0.2808	0	0.0897	–	0
λ_7	0.7667	0.2961	0.6812	0.3311	0.49	0.6571	–

Δ_8	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0	0	0	0	0
λ_2	0	–	0	0	0	0	0
λ_3	0	0	–	0	0	0	0
λ_4	0	0	0	–	0	0	0
λ_5	0	0	0	0	–	0	0
λ_6	0.171	0.1921	0.1538	0.2691	0.0427	–	0.1982
λ_7	0	0	0	0	0	0	–

With the help of above Matrices 18A-18H, and by applying Equation (6), the computed comprehensive concordance index is shown in Matrix 19.

Matrix 19: Comprehensive Concordance Index Matrix Δ

Δ	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.2419	0.3051	0.247	0.3116	0.2668	0.2905
λ_2	0.2133	–	0.3307	0.2377	0.3376	0.2832	0.3044
λ_3	0.133	0.1599	–	0.2085	0.2074	0.2188	0.1585
λ_4	0.4019	0.3953	0.5701	–	0.578	0.4243	0.6113
λ_5	0.1509	0.1568	0.2367	0.2101	–	0.2329	0.185
λ_6	0.1437	0.1555	0.269	0.22	0.2947	–	0.4242
λ_7	0.1938	0.1808	0.2612	0.2596	0.2433	0.2933	–

Step 12: Calculation of discordance indices. The partial discordance matrices shown in Matrices 20A-20H are calculated using Equation (8).

Matrices 20A, 20B: Discordance Indices w.r.t. First and Second Criteria

∇_1	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.4019	0.4394	1	0.1722	0.8908	0.0854
λ_2	0.6412	–	0.5591	1	0.2918	1	0.205
λ_3	0.6036	0.484	–	1	0.2543	0.9729	0.1675
λ_4	0	0	0	–	0	0.3065	0
λ_5	0.8709	0.7513	0.7888	1	–	1	0.4347
λ_6	0.1523	0.0327	0.0702	0.7366	0	–	0
λ_7	0.9577	0.8381	0.8756	1	0.6083	1	–

∇_2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.2476	0	0.8803	0.1574	0.4075	0
λ_2	0.1987	–	0	0.8559	0.133	0.3831	0
λ_3	0.5742	0.5986	–	1	0.5085	0.7586	0.0114
λ_4	0	0	0	–	0	0	0
λ_5	0.2889	0.3133	0	0.9461	–	0.4733	0
λ_6	0.0388	0.0632	0	0.6959	0	–	0
λ_7	0.786	0.8104	0.4349	1	0.7202	0.9704	–

Matrices 20C, 20D: Discordance Indices w.r.t. Third and Fourth Criteria

∇_3	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0	0	0	0	0
λ_2	0.4038	–	0	0	0	0	0.3283
λ_3	0.3008	0	–	0	0	0	0.2252
λ_4	0.5178	0	0	–	0	0	0.4423
λ_5	0.4193	0	0	0	–	0	0.3437
λ_6	0.3022	0	0	0	0	–	0.2266
λ_7	0	0	0	0	0	0	–

∇_4	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.4557	0.0628	1	0	0.0311	0
λ_2	0.0961	–	0	1	0	0	0
λ_3	0.489	0.6688	–	1	0	0.2442	0.1116
λ_4	0	0	0	–	0	0	0
λ_5	0.7703	0.95	0.5571	1	–	0.5254	0.3928
λ_6	0.5207	0.7005	0.3076	1	0.0264	–	0.1433
λ_7	0.6534	0.8331	0.4402	1	0.159	0.4085	–

Matrices 20E, 20F: Discordance Indices w.r.t. Fifth and Sixth Criteria

∇_5	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0	0	0	0	0
λ_2	0	–	0	0	0	0	0
λ_3	0	0	–	0	0	0	0
$< /p >< p > \lambda_4$	0.3632	0.0187	0	–	0	0	0
λ_5	0	0	0	0	–	0	0
λ_6	0.2079	0	0	0	0	–	0
λ_7	0.3113	0	0	0	0	0	–

∇_6	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0	0	0.6135	0	0.369	0
λ_2	0	–	0	0.3303	0	0.0859	0
λ_3	0	0	–	0.3681	0	0.1237	0
λ_4	0	0	0	–	0	0	0
λ_5	0	0.1191	0.0813	0.8457	–	0.6013	0.024
λ_6	0	0	0	0	0	–	0
λ_7	0	0	0	0.4254	0	0.181	–

Matrices 20G, 20H: Discordance Indices w.r.t. Seventh and Eighth Criteria

∇_7	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.2349	0	0.1965	0.0219	0	0.2782
λ_2	0	–	0	0	0	0	0
λ_3	0	0.141	–	0.1026	0	0	0.1843
λ_4	0	0	0	–	0	0	0
λ_5	0	0	0	0	–	0	0
λ_6	0	0.1145	0	0.0761	0	–	0.1578
λ_7	0	0	0	0	0	0	–

∇_8	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.2061	0.237	0.1442	0.3264	0.5839	0.2012
λ_2	0.2402	–	0.254	0.1612	0.3435	0.601	0.2182
λ_3	0.2093	0.1923	–	0.1303	0.3126	0.5701	0.1874
λ_4	0.3021	0.2851	0.316	–	0.4054	0.6629	0.2802
λ_5	0.1199	0.1029	0.1337	0.0409	–	0.4807	0.0979
λ_6	0	0	0	0	0	–	0
λ_7	0.2451	0.2281	0.2589	0.1661	0.3484	0.6059	–

Step 13: Calculation of credibility index. The credibility index shown in Matrix 21 is calculated using Equation (9).

Matrix 21: Credibility Index

π	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	–	0.136	0.2461	0	0.3049	0.0157	0.2905
λ_2	0.0712	–	0.2178	0	0.3346	0	0.2939
λ_3	0.0129	0.0178	–	0	0.1049	0.0012	0.1351
λ_4	0.324	0.3953	0.5701	–	0.578	0.2484	0.6113
λ_5	0.0036	0.0022	0.038	0	–	0	0.077
λ_6	0.06	0.0551	0.2548	0	0.2947	–	0.4242
λ_7	0.0009	0.0016	0.0255	0	0.0401	0	–

Lastly, the last step comes under Module II: The Exploitation of Outranking Relations:

Step 14: Calculation of net credibility index and ranking. The net credibility index and ranking of alternatives shown in Table *6* is calculated using Equations (10)-(12).

Alternatives	μ^+	μ^-	μ	Ranking
Banda λ_1	0.9932	0.4726	0.5206	3
Chitrakoot λ_2	0.9175	0.6080	0.3095	4
Hamirpur λ_3	0.2719	1.3523	−1.0804	5
Jalaun λ_4	2.7271	0	2.7271	1
Jhansi λ_5	0.1208	1.6572	−1.5364	6
Lalitpur λ_6	1.0888	0.2653	0.8235	2
Mahoba λ_7	0.0681	1.8320	−1.7639	7

TABLE 6: Exploitation of Outranking Relations and Final Ranking

$$\lambda_4 \succ \lambda_6 \succ \lambda_1 \succ \lambda_2 \succ \lambda_3 \succ \lambda_5 \succ \lambda_7$$

$$\text{Jalaun} \succ \text{Lalitpur} \succ \text{Banda} \succ \text{Chitrakoot} \succ \text{Hamirpur} \succ \text{Jhansi} \succ \text{Mahoba}$$

Thus, District Jalaun is the best location in Bundelkhand region of Uttar Pradesh for solar power plant/park installation.

The ranking indicates that Jalaun performs consistently well across the major positive criteria such as abundant solar radiation, flat open land availability, and proximity to transmission lines. It also shows comparatively lower negative impact values for extreme weather conditions and dust density. Relative to the other districts, Jalaun achieves higher aggregated concordance scores and faces fewer veto or discordance effects. These combined strengths lead to its highest net credibility index in the PN-ELECTRE-III evaluation, making it the most suitable location among the seven alternatives.

Conclusions

In conclusion, the research presents a novel approach to solving the solar plant location problem in India. The use of the ELECTRE III method in the PNP approach is a significant contribution to the field of decision-making under uncertainty. The paper's two-phase decision-aiding system allows for a comprehensive evaluation of potential solar plant locations, taking into account multiple criteria and stakeholder preferences. The case study on green energy in India demonstrates the effectiveness of the proposed method in identifying the most suitable locations for solar plants, considering economic, social, and environmental factors. The present study contributes a novel integration of the ELECTRE-III method with the PNP framework, which enables decision-makers to model uncertainty, indeterminacy, and conflicting information more effectively than traditional crisp or fuzzy MCDM approaches. Unlike classical ELECTRE-III, the proposed PN-ELECTRE-III system incorporates PNNs to reflect expert hesitation and data ambiguity, offering a more realistic representation of real-world solar site selection conditions. The significance of this work lies in its structured two-phase methodology that improves transparency, consistency, and robustness in renewable-energy planning.

However, this study also has limitations. The results rely on expert-assigned linguistic judgments, which may introduce subjectivity. The case study is limited to the Bundelkhand region, so generalizability to other regions may vary depending on climate, geography, and policy constraints. Additionally, computational complexity increases as the number of criteria or alternatives grows. Future research may extend this work by validating the model with larger datasets, integrating GIS-based spatial analysis, comparing PN-ELECTRE-III with other neutrosophic or hybrid MCDM methods, or developing software tools that automate the full decision-aiding process. Incorporating real-time solar irradiance data and economic forecasting models may further enhance decision accuracy. Overall, the paper highlights the importance of using advanced decision-making tools to address complex problems in renewable energy development. The proposed method can serve as a valuable resource for policymakers, investors, and stakeholders involved in the planning and implementation of solar energy projects in India and other countries facing similar challenges.

Appendices

The article was previously published as a preprint available in the following link: https://philpapers.org/archive/SAISPP-2.pdf

Data Availability

The datasets and code generated and/or analyzed during this study are included in this article or are available from the corresponding author upon reasonable request.

Additional Information

Author Contributions

All authors have reviewed the final version to be published and agreed to be accountable for all aspects of the work.

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Disclosures

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